Refinement of Green's function method for parameter tuning

Tsuyoshi Wakamatsu [†‡]
Michael G.G. Foreman [†‡]

† University of Victoria, ‡ DFO Canada

OVERVIEW:

- Purpose: Tuning model parameters against data
- Method: Green's Function method (Menemenlis et al. 2005) + stabilization
- Testbed : twin experiments with NEMO GYRE01/12 configurations

GOAPP
Global Ocean-Atmosphere Prediction and Predictability

Fisheries and Oceans Canada
Pêches et Océans Canada

Background: 4D-Var formulation

Ocean Circulation Model:

$$\mathbf{y}_{\mathrm{model}} = \boldsymbol{G}[\mathbf{p}]$$

e,g,) SLA, temperature, salinity...

where

$$\mathbf{p} = [\mathrm{p}_1, \ldots, \mathrm{p}_N]$$

e,g,) transfer coefficients, eddy diffusivity/viscosity ...

Cost Function:

$$\mathrm{J}[\mathbf{p}] = (\boldsymbol{G}[\mathbf{p}] - \mathbf{y}_{\mathrm{data}})^{\mathrm{T}} \mathbf{R}^{-1} (\boldsymbol{G}[\mathbf{p}] - \mathbf{y}_{\mathrm{data}})$$

Normal Equation in Incremental Form:

$$\mathbf{G}^{\mathrm{T}} \mathbf{R}^{-1} (\mathbf{G}\,\delta\mathbf{p} - \mathbf{d}) = 0, \quad \mathbf{G} = \partial \boldsymbol{G} / \partial \mathbf{p}$$

where

$$\mathbf{d} = \boldsymbol{G}[\mathbf{p}^{\mathrm{b}}] - \mathbf{y}_{\mathrm{data}}, \quad \delta\mathbf{p} = \mathbf{p} - \mathbf{p}^{\mathrm{b}}$$

Best Estimate:

$$\delta\mathbf{p}^{\mathrm{a}} = (\mathbf{G}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{G})^{-1} \mathbf{G}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{d}$$

Background: Green's Function method

Best Estimate:

$$\delta \mathbf{p}^{a} = (\mathbf{G}^{T}\mathbf{R}^{-1}\mathbf{G})^{-1}\mathbf{G}^{T}\mathbf{R}^{-1}\mathbf{d}$$

Data Kernel:

$$\mathbf{G} = [\mathbf{g}_1, \ldots, \mathbf{g}_N]$$

Green's functions $(g_1,\ldots,g_N)$ are approximated by finite differences of model outputs

Green's Functions:

$$\mathbf{g}_n = \Delta \mathbf{y}_n / \Delta p_n$$

where

$$\Delta \mathbf{y}_n = \boldsymbol{G}[\mathbf{p}^{b} + \mathbf{e}_n \Delta p_n] - \boldsymbol{G}[\mathbf{p}^{b}]$$

Due to smallness of Δp, the above formula could become numerically unstable.

Linear System:

$$\mathbf{A}\delta \mathbf{p} = \mathbf{b}, \quad \mathbf{A} = \mathbf{G}^{T}\mathbf{R}^{-1}\mathbf{G}, \quad \mathbf{b} = \mathbf{G}^{T}\mathbf{R}^{-1}\mathbf{d}$$

$$A_{nn} = \Delta \mathbf{y}_n^{T} \Delta \mathbf{y}_n \sigma_n^{-2} \Delta p_n^{-2}$$

Stabilized Green's Function method

Operation of division by Δp can be completely removed from the system

Best Estimate:

$$\delta \mathbf{p}^{a} = \mathbf{P}\left(\mathbf{D}^{T}\mathbf{R}^{-1}\mathbf{D}\right)^{-1}\mathbf{D}^{T}\mathbf{R}^{-1}\mathbf{d}$$

where

$$\mathbf{D} = \left[\Delta \mathbf{y}_{1}, \ldots, \Delta \mathbf{y}_{N}\right]$$

$$\mathbf{P} = \mathrm{diag}\left(\Delta p_{1}, \ldots, \Delta p_{N}\right)$$

Stabilized Green's function method

Linear System:

$$\tilde{\mathbf{A}}\mathbf{x} = \tilde{\mathbf{b}}, \quad \tilde{\mathbf{A}} = \mathbf{D}^{T}\mathbf{R}^{-1}\mathbf{D}, \quad \tilde{\mathbf{b}} = \mathbf{D}^{T}\mathbf{R}^{-1}\mathbf{d}$$

$$\tilde{A}_{nn} = \Delta \mathbf{y}_{n}^{T}\Delta \mathbf{y}_{n}\sigma_{n}^{-2}$$

Best Estimate:

$$\delta \mathbf{p}^{a} = \mathbf{P}\mathbf{x}^{a}$$

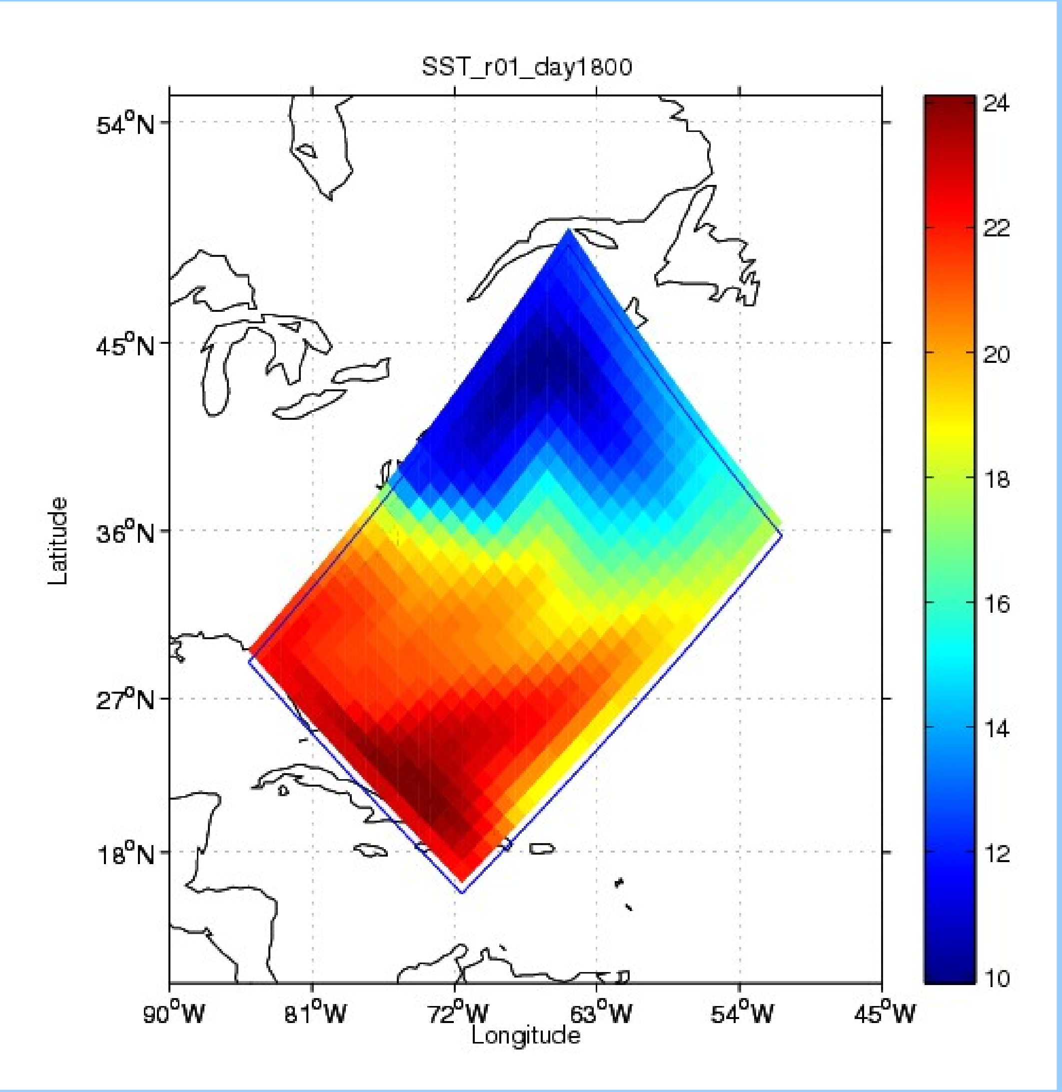
Testbed: NEMO GYRE01 configuration

- 1 degree GYRE configuration (Hazeleger and Drijfhout, 2000)

- seasonal, zonally uniform, analytical forcing

- 200 years spinning-up run
  + 10 years for model calibration

- 200 years spinning-up run
  + 10 years run for simulated data
  (identical twin experiments)

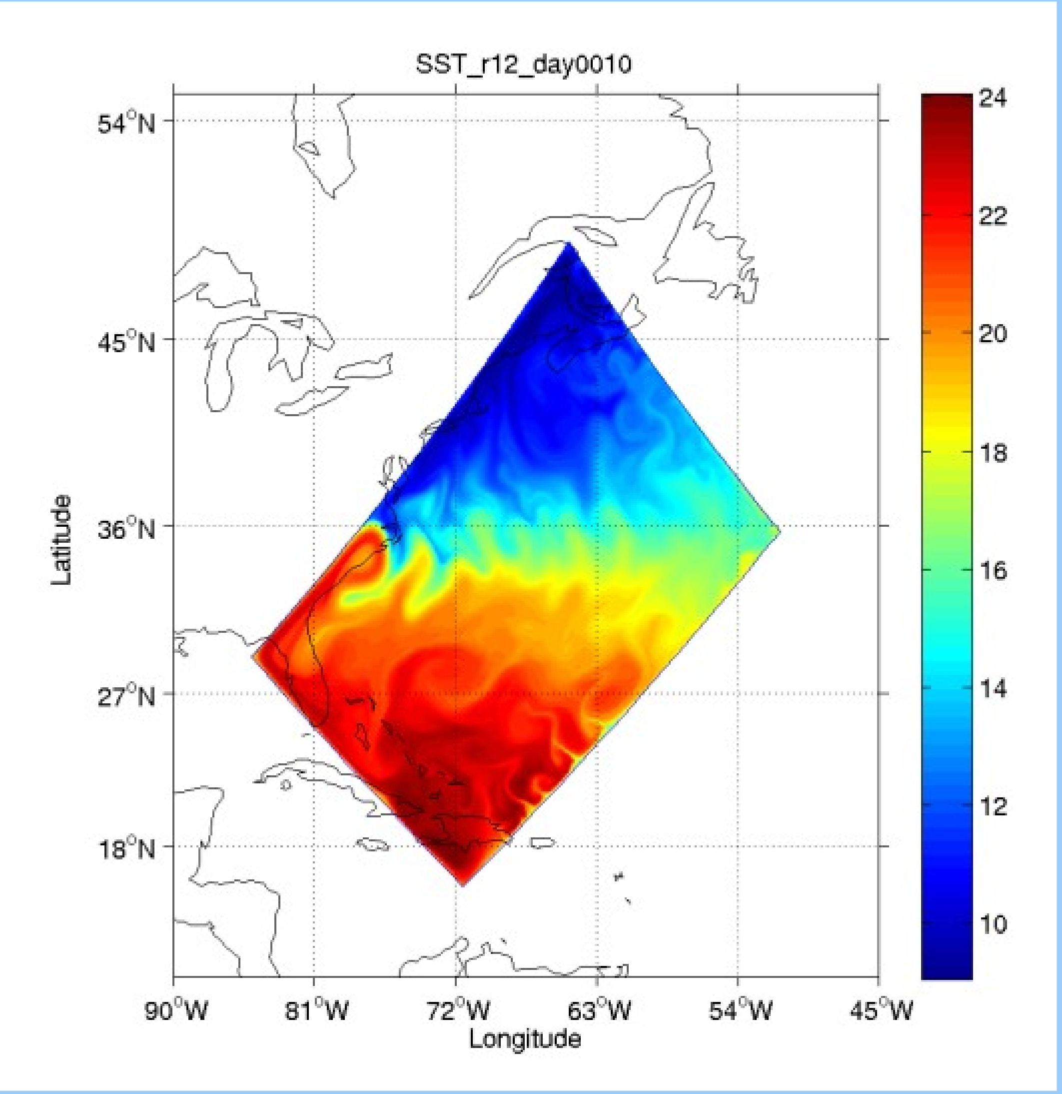


Testbed: NEMO GYRE12 configuration

- 1/12 degree GYRE configuration (Hazeleger and Drijfhout, 2000)

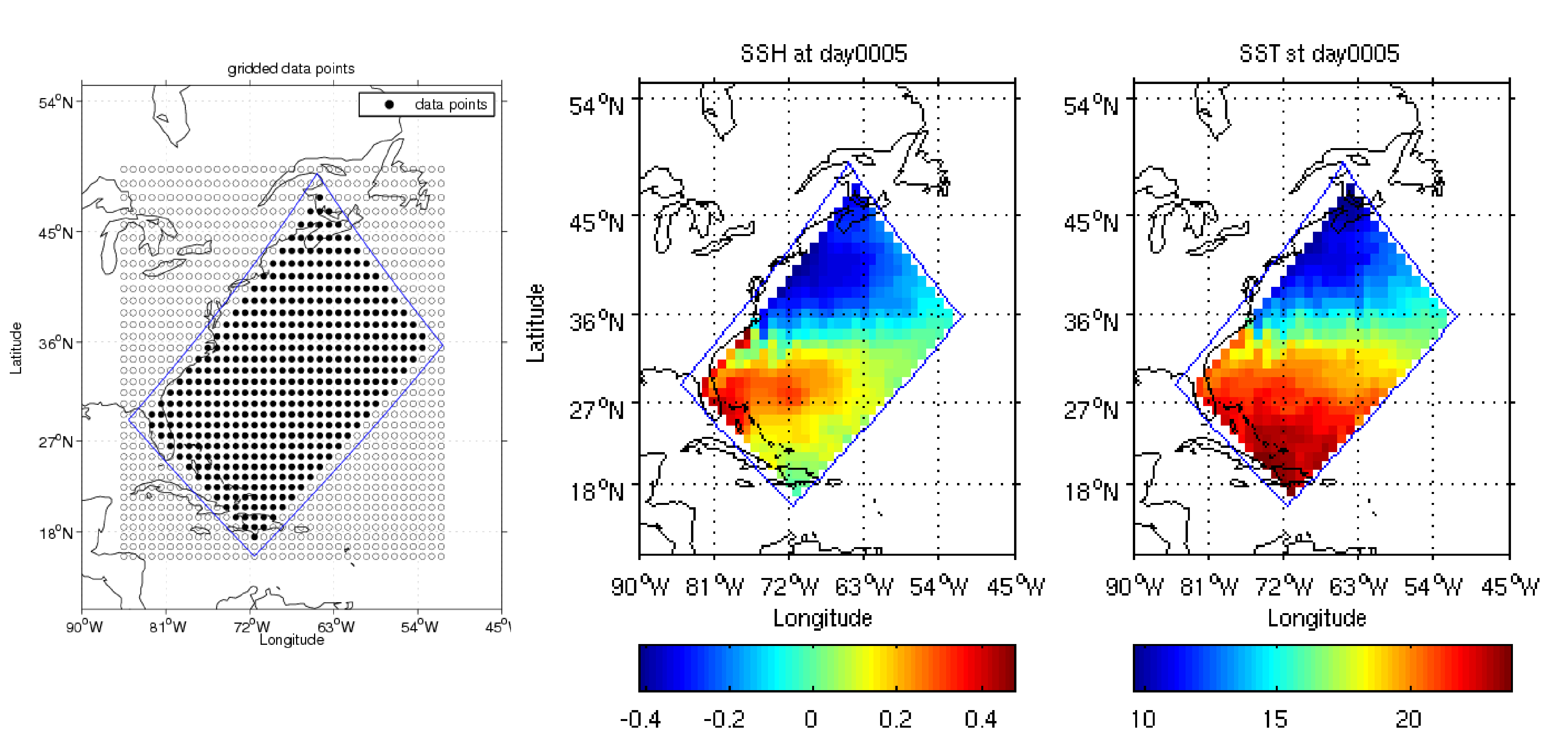
- seasonal, zonally uniform, analytical forcing

- 200 years GYRE01 spinning-up run
  + 10 years GYRE12 spinning-up run
  + 5 years run for simulated data
    (fraternal twin experiments)



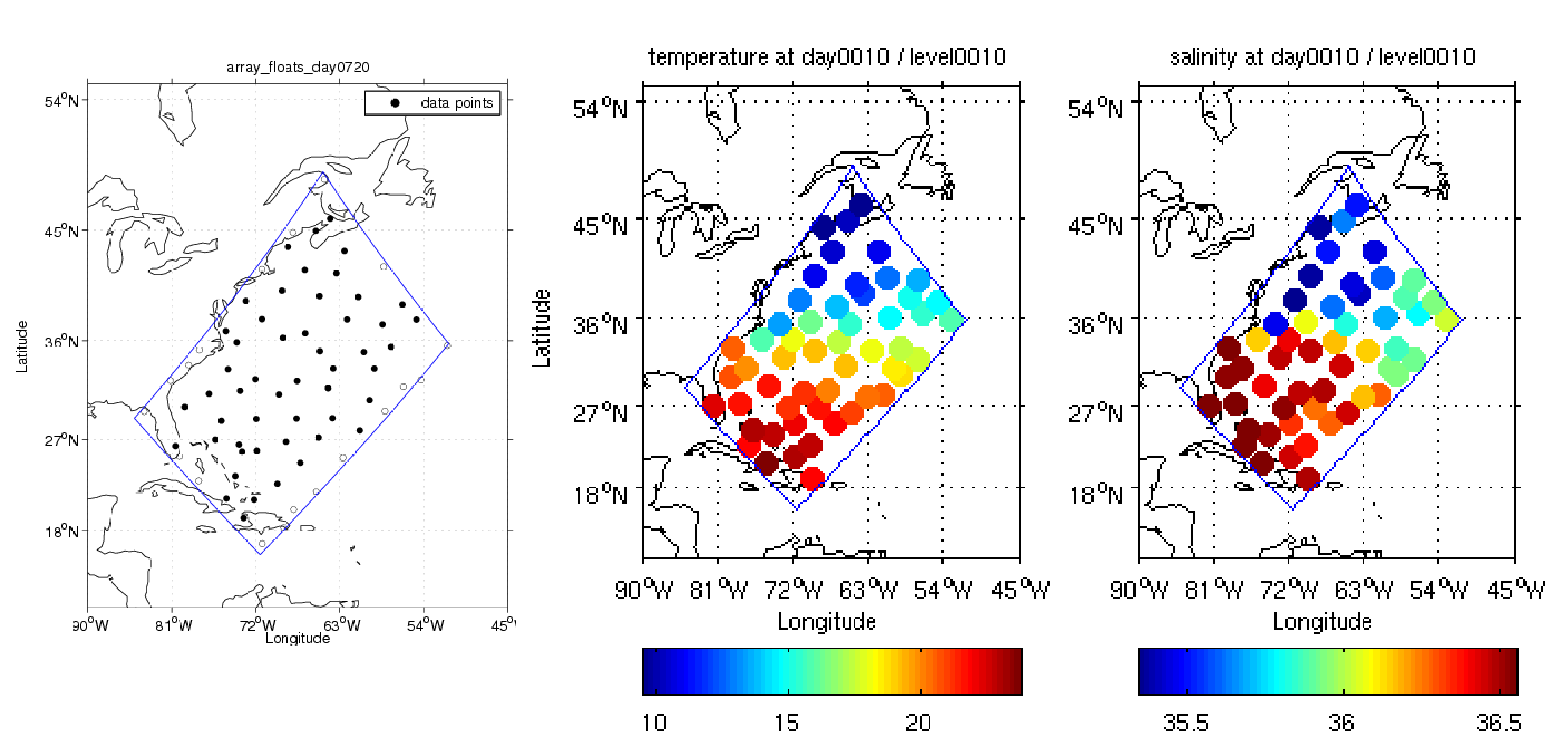
Testbed: Simulated Data

- 1x1 gridded surface (satellite) data: SSH and SST (10 days mean)



Testbed: Simulated Data

- 3x3 Argo float data: potential temperature and salinity
  (every 10 days, Levitus94 standard depth (>2000m) + enhanced resolution in upper 50m)



Identical twin experiments

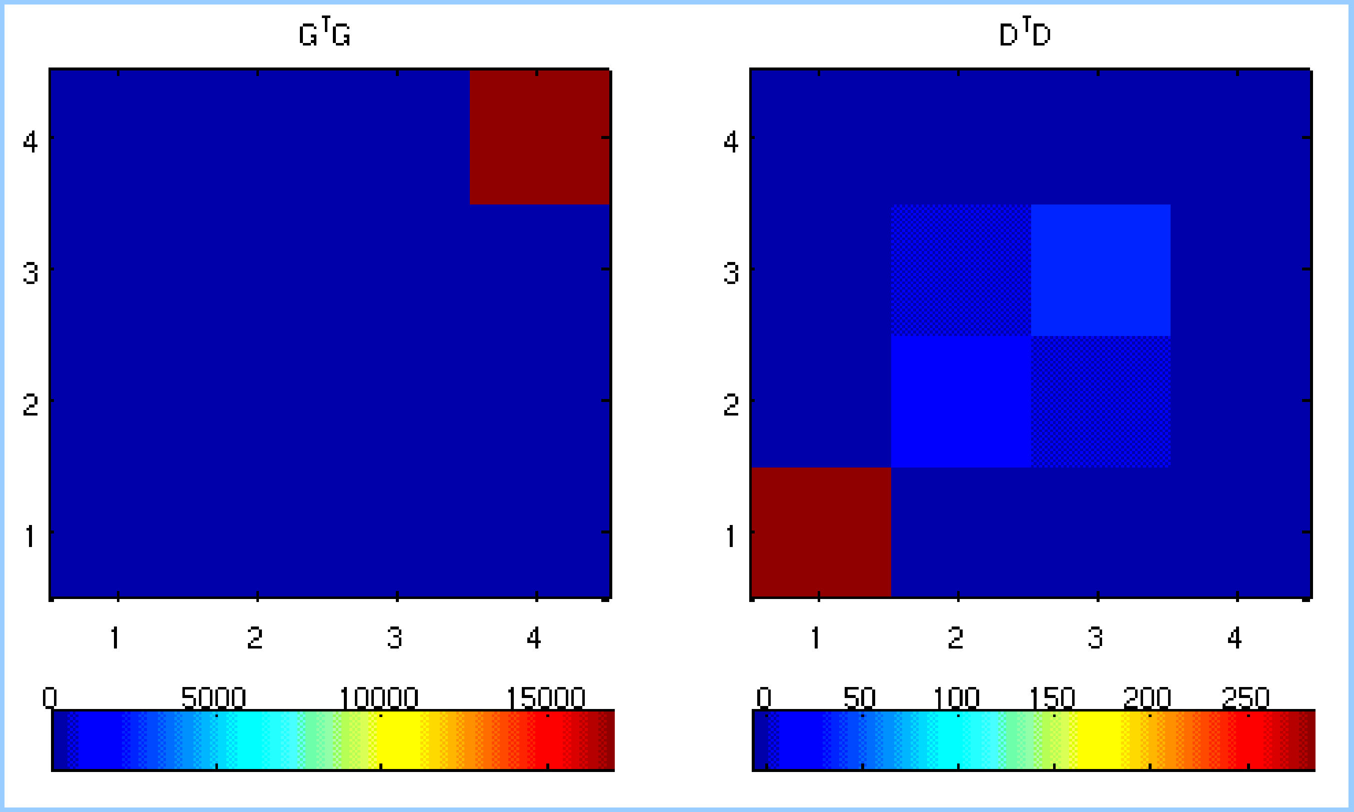
- assimilation window: 100days-10 years
- data: SST, SSH, temperature, salinity from GYRE01 configuration
- perfect initial condition
- control parameters:

	parameter	unit	p^t	p^b	Δp	p^a(100day)	p^a(5year)
1	SST relaxation coefficient	$Wm^{-2}K^{-1}$	-60	-30	-10		
2	horizontal eddy diffusivity	m^2s^{-1}	1.5e3	0.75e3	0.25e3		
3	horizontal eddy viscosity	m^2s^{-1}	1.0e5	0.75e5	0.25e5		
4	bottom drag coefficient		1.5e-3	0.75e-3	0.25e-3		

- Green's function method:

  - R = I
  - linear system: Ax = b

$A = G^TG$: Green's Function (GF)
$A = D^TD$: Stabilized GF



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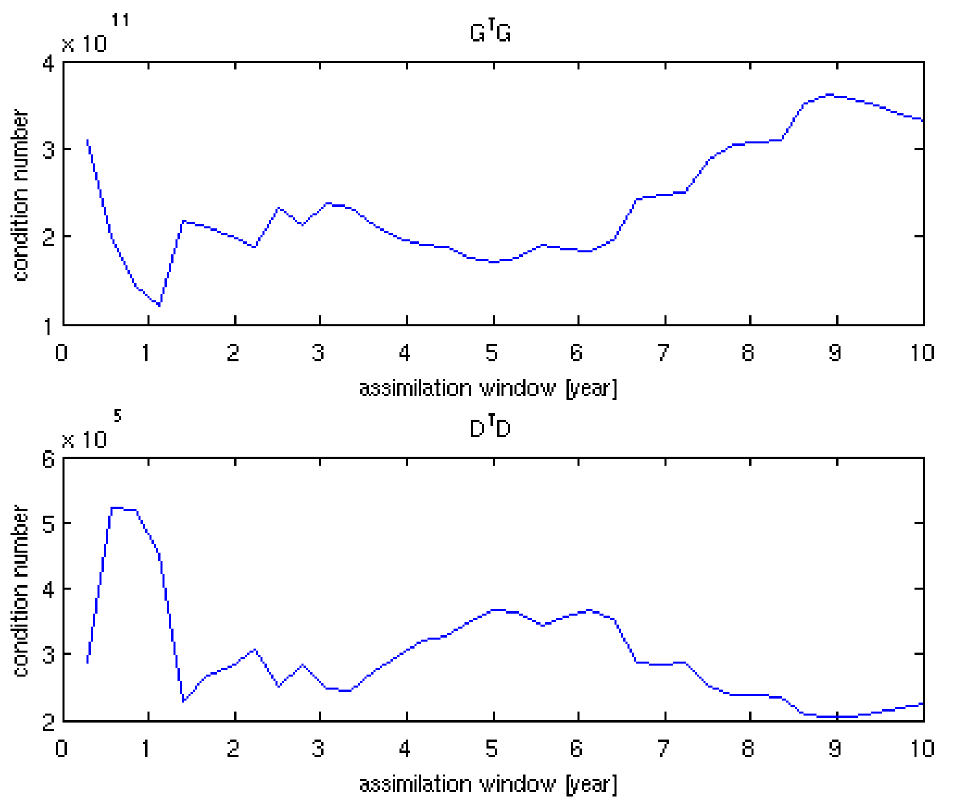
  - condition number



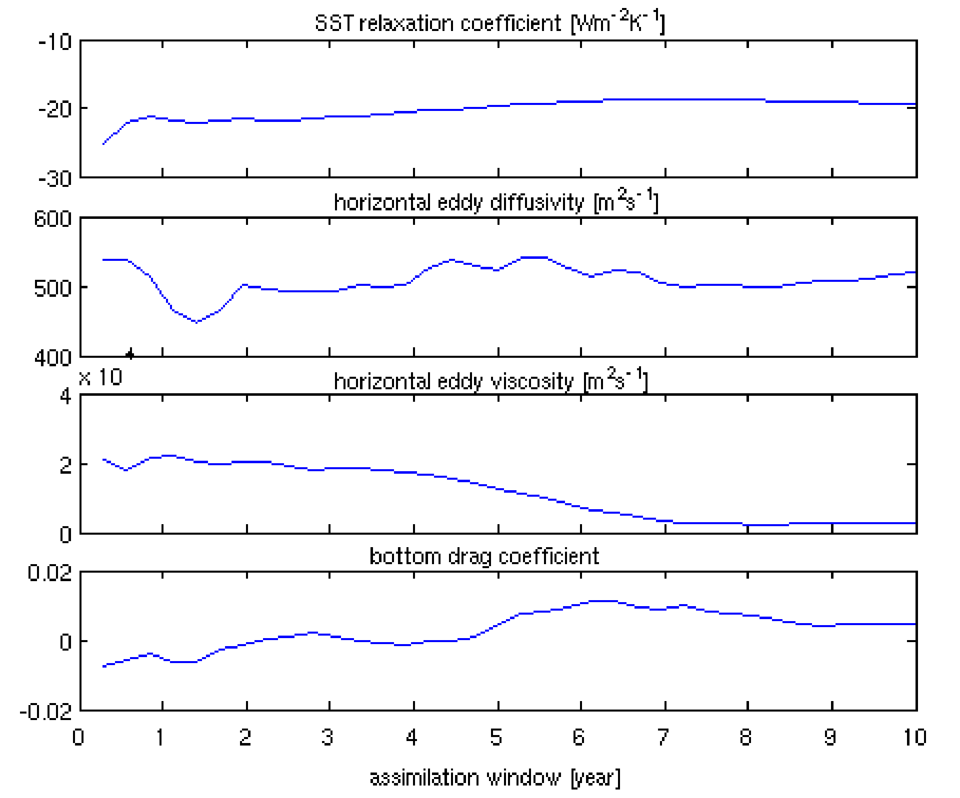
Identical twin experiments

- assimilation window: 100days-10 years
- data: SST, SSH, temperature, salinity from GYRE01 configuration
- perfect initial condition
- control parameters:

	parameter	unit	p^t	p^b	Δp	p^a(100day)	p^a(5year)
1	SST relaxation coefficient	$Wm^{-2}K^{-1}$	-60	-30	-10	-55.1	-49.7
2	horizontal eddy diffusivity	m^2s^{-1}	1.5e3	0.75e3	0.25e3	1.29e3	1.27e3
3	horizontal eddy viscosity	m^2s^{-1}	1.0e5	0.75e5	0.25e5	0.96e5	0.86e5
4	bottom drag coefficient		1.5e-3	0.75e-3	0.25e-3	-6.95e-3	5.45e-3

- Green's function method
- optimal estimation

Δp^a



Fraternal twin experiments

- assimilation window: 100days-5 years
- data: SST, SSH, temperature, salinity from GYRE12 configuration
- imperfect initial condition (model climatology from GYRE12)
- control parameters:

	parameter	unit	p^b	Δp	p^a(100day)	p^a(5year)
1	SST relaxation coefficient	$Wm^{-2}K^{-1}$	-30	-10		
2	horizontal eddy diffusivity	m^2s^{-1}	0.75e3	0.25e3		
3	horizontal eddy viscosity	m^2s^{-1}	0.75e5	0.25e5		
4	bottom drag coefficient		0.75e-3	0.25e-3		

- Green's function method:
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  - linear system: Ax = b

A = G^TG: Green's Function (GF)
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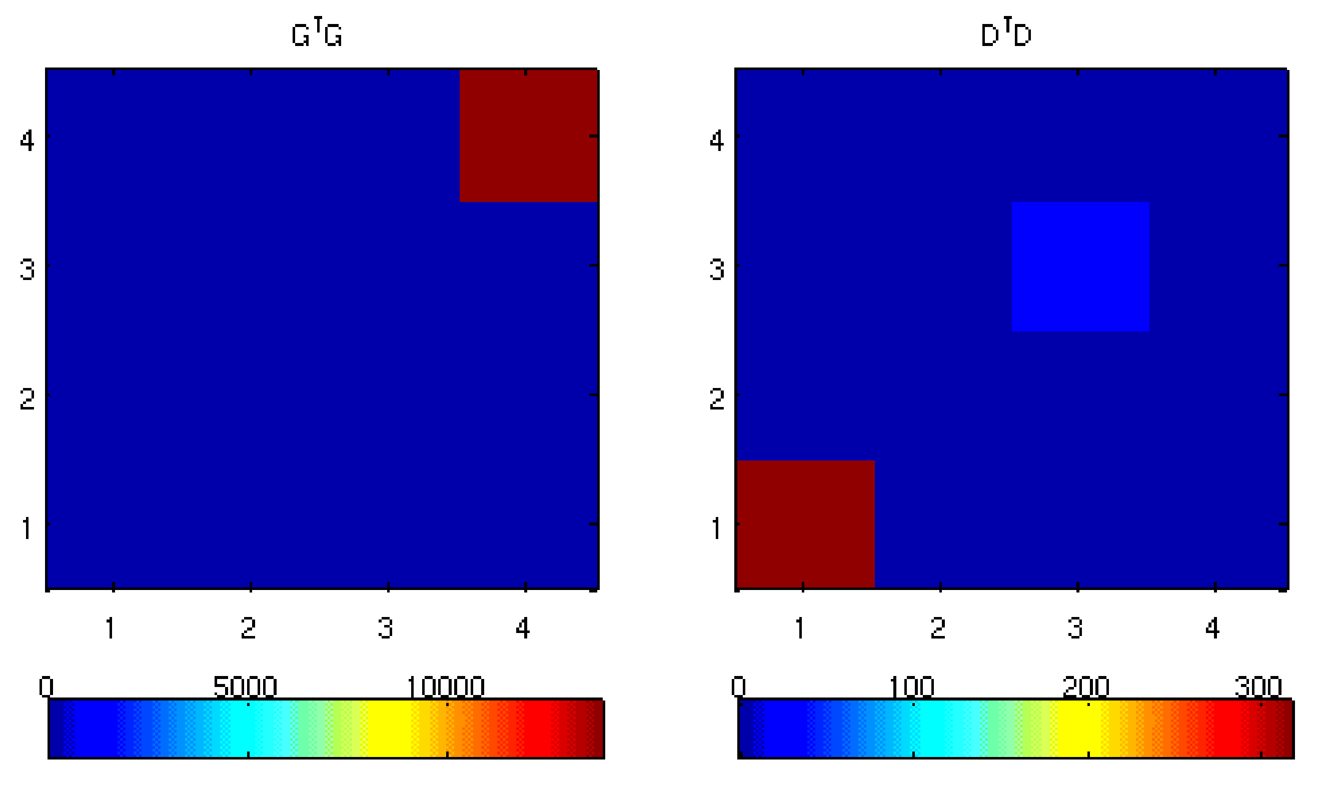

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- assimilation window: 100days-5 years
- data: SST, SSH, temperature, salinity from GYRE12 configuration
- imperfect initial condition (model climatology from GYRE12)
- control parameters:

	parameter	unit	p^b	Δp	p^a(100day)	p^a(5year)
1	SST relaxation coefficient	$Wm^{-2}K^{-1}$	-30	-10		
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3	horizontal eddy viscosity	m^2s^{-1}	0.75e5	0.25e5		
4	bottom drag coefficient		0.75e-3	0.25e-3		

- Green's function method:

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    $A = G^TG$: Green's Function (GF)
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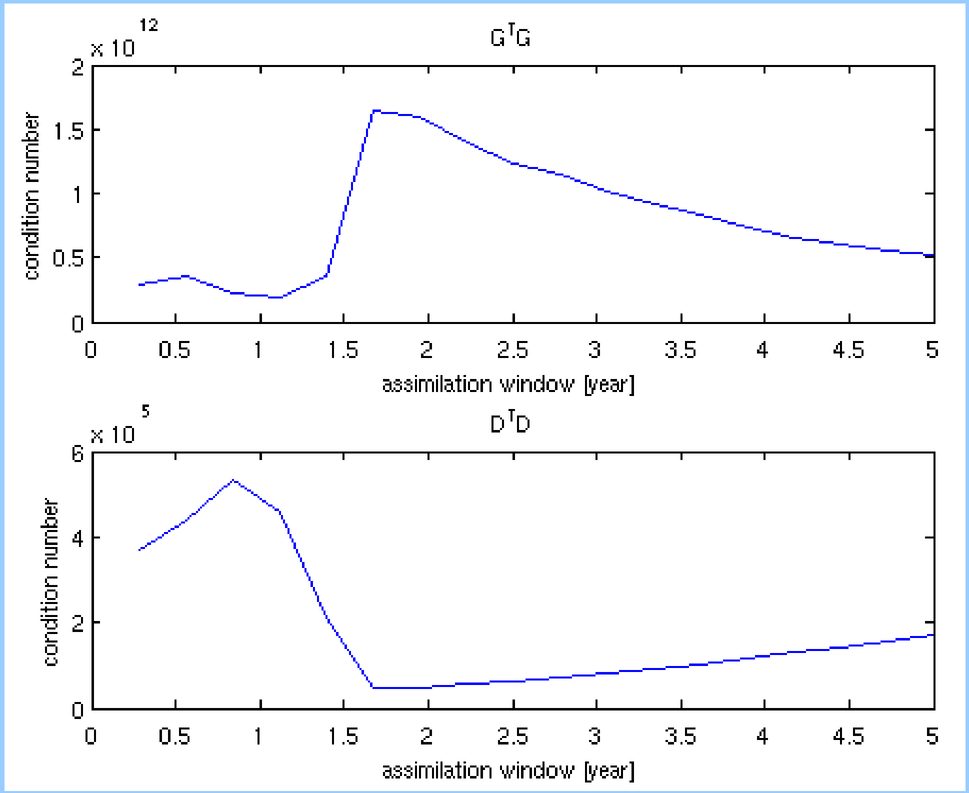
  - condition number



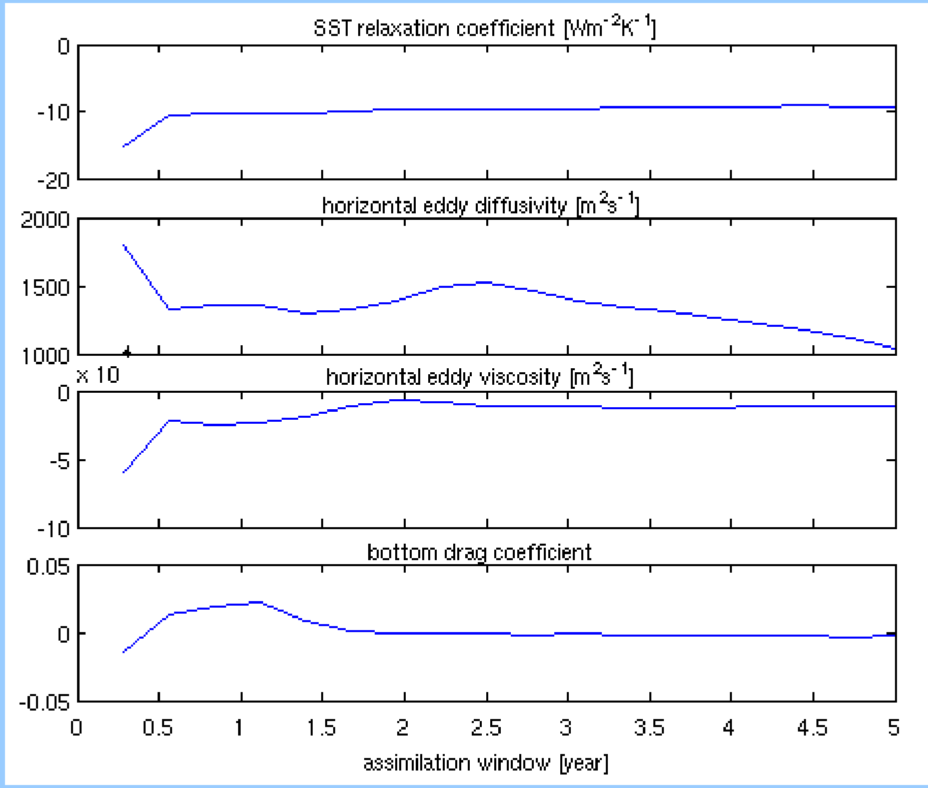
Fraternal twin experiments

- assimilation window: 100days-5 years
- data: SST, SSH, temperature, salinity from GYRE12 configuration
- imperfect initial condition (model climatology from GYRE12)
- control parameters:

	parameter	unit	p^b	Δp	p^a(100day)	p^a(5year)
1	SST relaxation coefficient	$Wm^{-2}K^{-1}$	-30	-10	-45.2	-39.3
2	horizontal eddy diffusivity	m^2s^{-1}	0.75e3	0.25e3	2.54e3	1.79e3
3	horizontal eddy viscosity	m^2s^{-1}	0.75e5	0.25e5	0.15e5	0.64e5
4	bottom drag coefficient		0.75e-3	0.25e-3	-14.3e-3	-1.85e-3

- Green's function method
- Optimal estimation

Δp^a



Fraternal twin experiments

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- data: SST, SSH, temperature, salinity from GYRE12 configuration
- imperfect initial condition (model climatology from GYRE12)
- control parameters:

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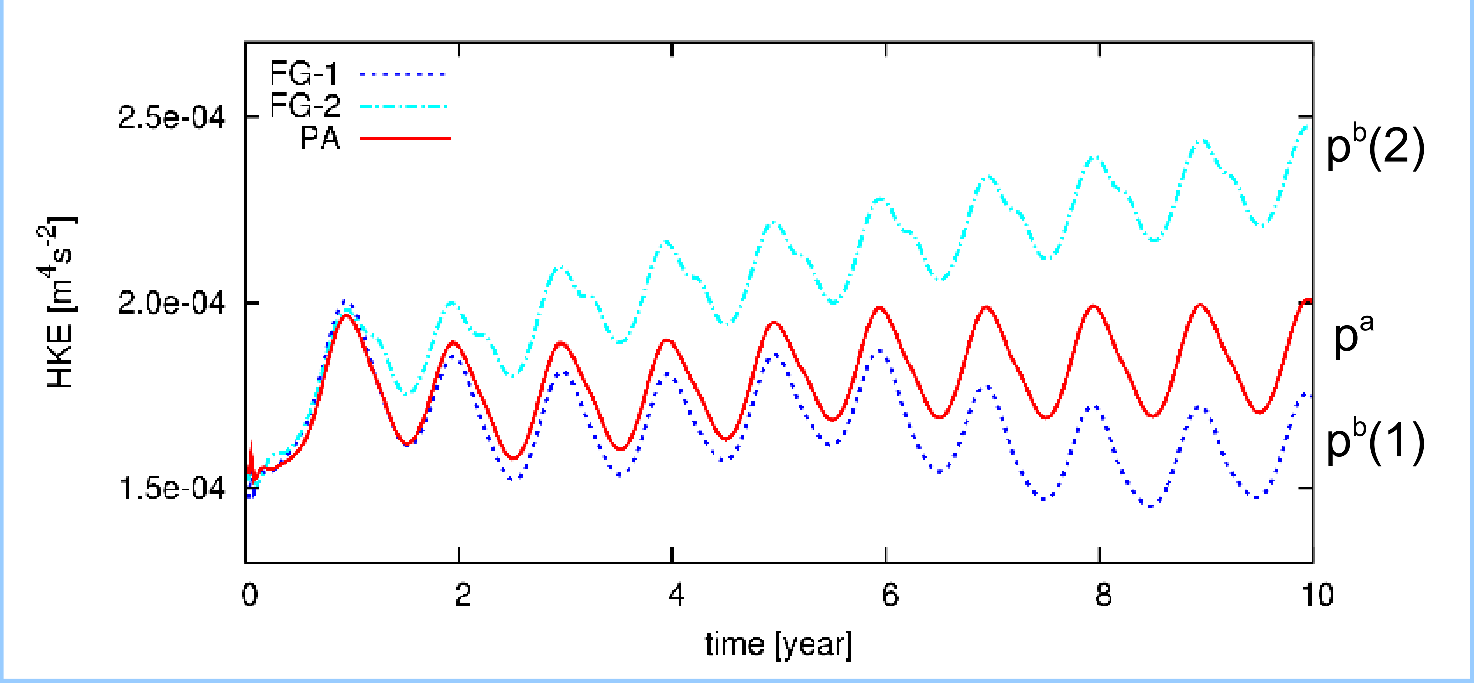
3	horizontal eddy viscosity	m^2s^{-1}	0.75e5	0.25e5	0.15e5	0.64e5
4	bottom drag coefficient		0.75e-3	0.25e-3	-14.3e-3	-1.85e-3

- Green's function method
- Optimal estimation
- Balance in model state



Summary

- Green's Function package for NEMO
  - numerically stabilized formulation
  - surface gridded data
  - Argo float data

- Testbed with NEMO GYRE configuration
  - identical Twin: GYRE01-GYRE01
  - fraternal Twin: GYRE01-GYRE12

- Future application
  - real data (SLA, SST, Argo, ...)
  - larger number of control parameters (coefficients in bulk formula, eddy mixing parametrization, model bias,...)

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