The singular vector (SV) analysis of an ENSO prediction model for the period from 1856-2003

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Introduction

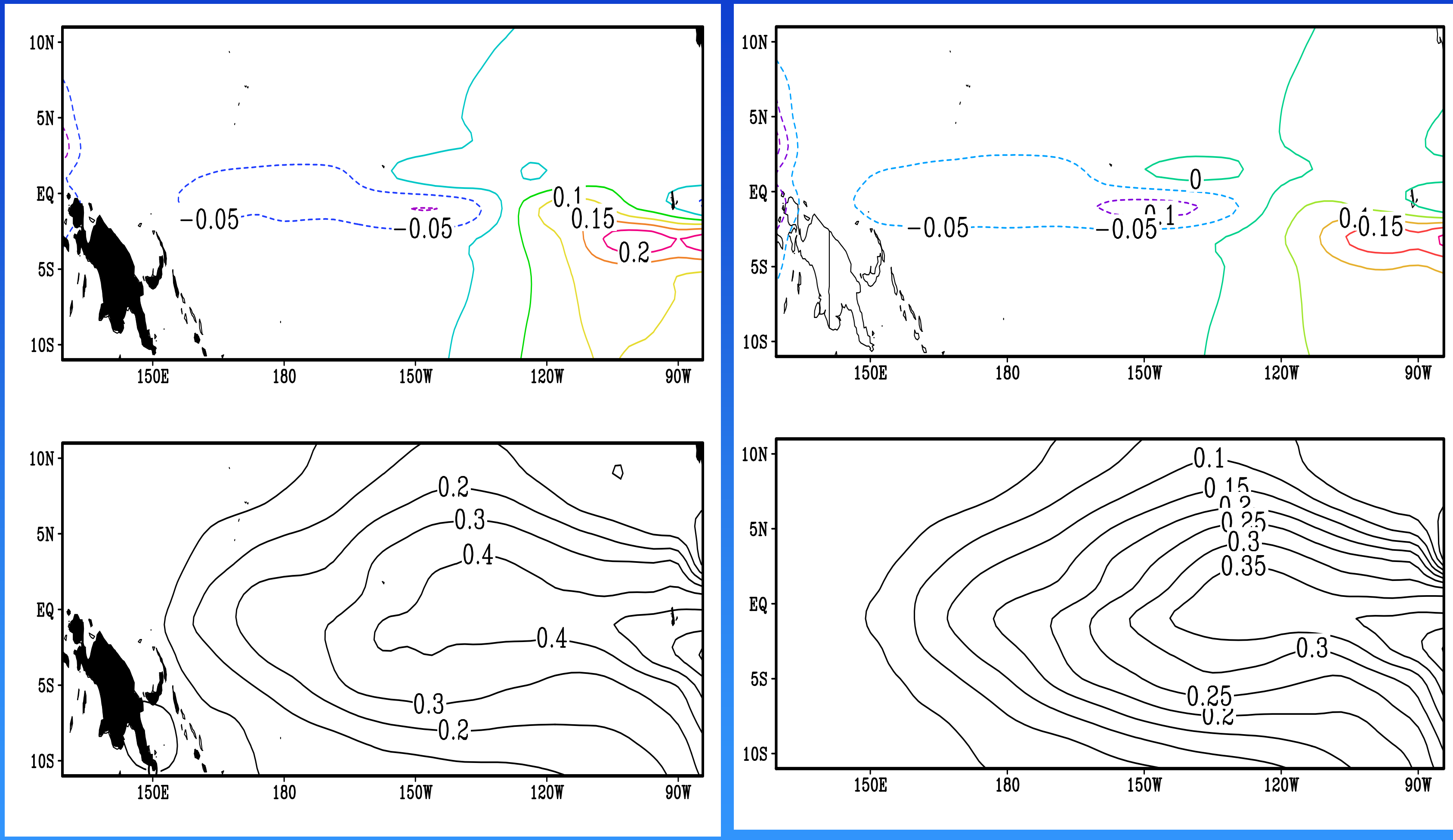
- Model initial error and predictability
- Singular vector analysis

1. **Construct** the tangent linear model by an Automatic Differentiation Engine http://tapenade.inria.fr:8080/tapenade/index.jsp
2. Obtain the optimal model error growth on the full TLM model in the real model physical space. The model initial SSTA field is perturbed for each model grid at each model run (Chen,1997) .
3. SVD analysis method to get the singular vector (SV), singular value, and final pattern.

The first Singular vector (SV) and Final pattern (FP) at 6-month leading time

SV

FP



Climatological run

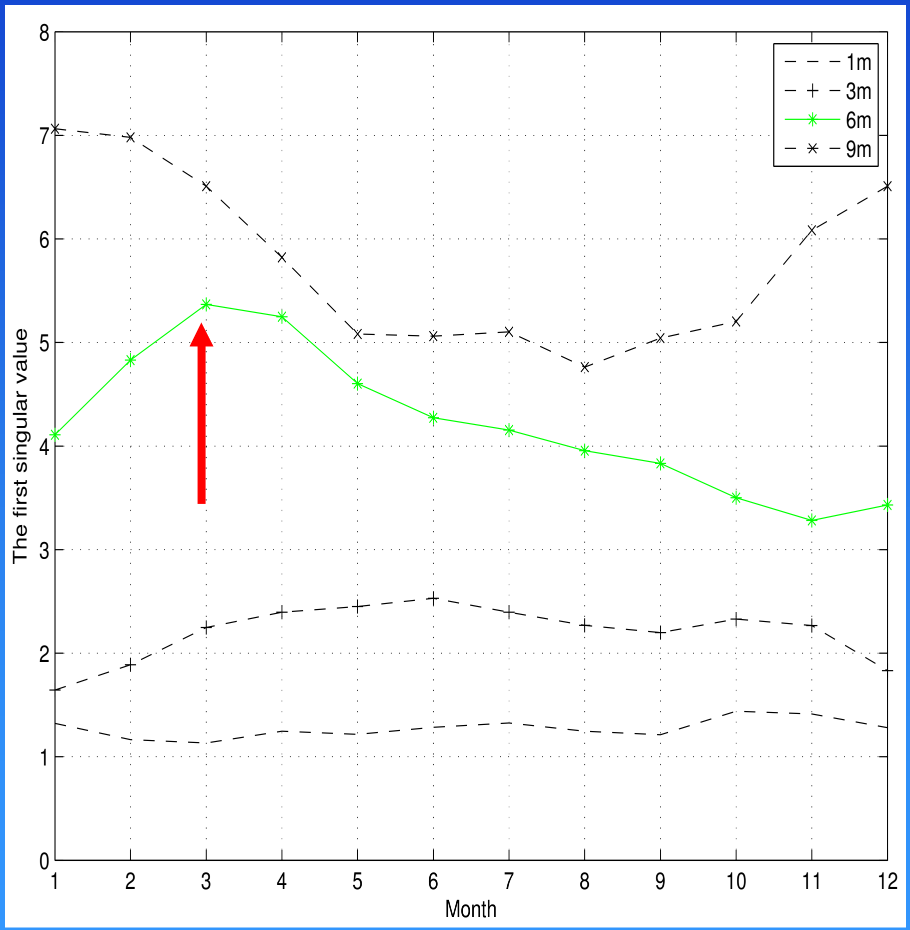
(only climatological seasonal mean fields are included)

148-year run

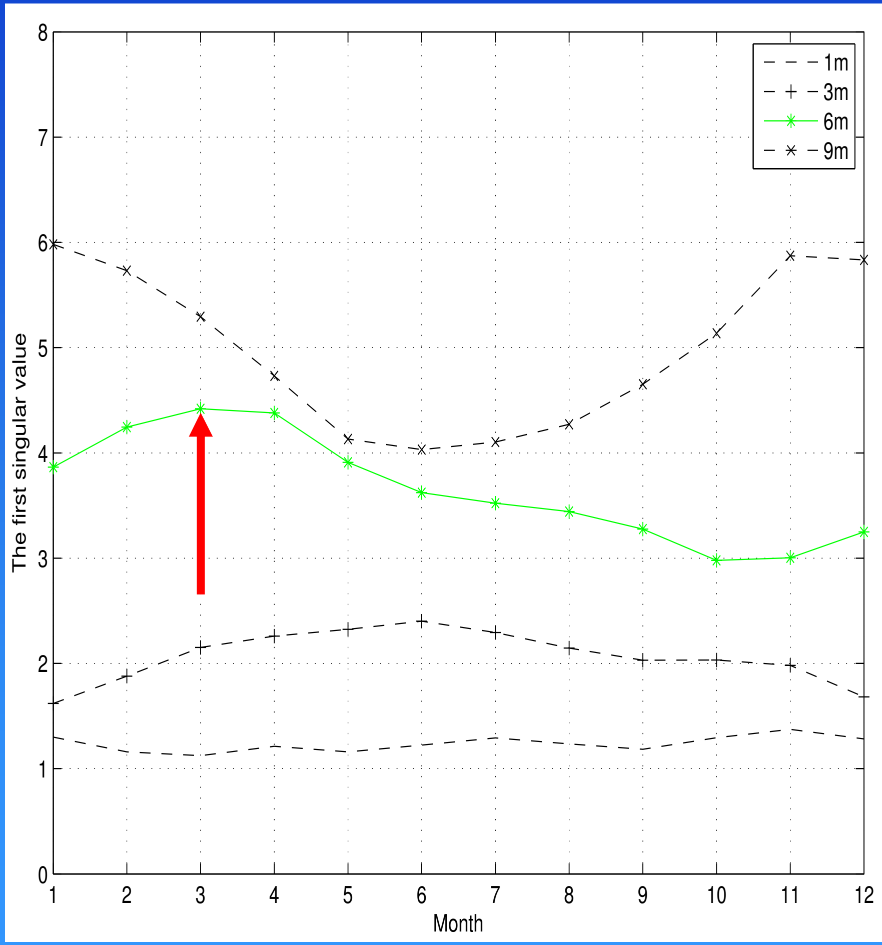
use the real initial SSTA

Singular Values

Climatological Run

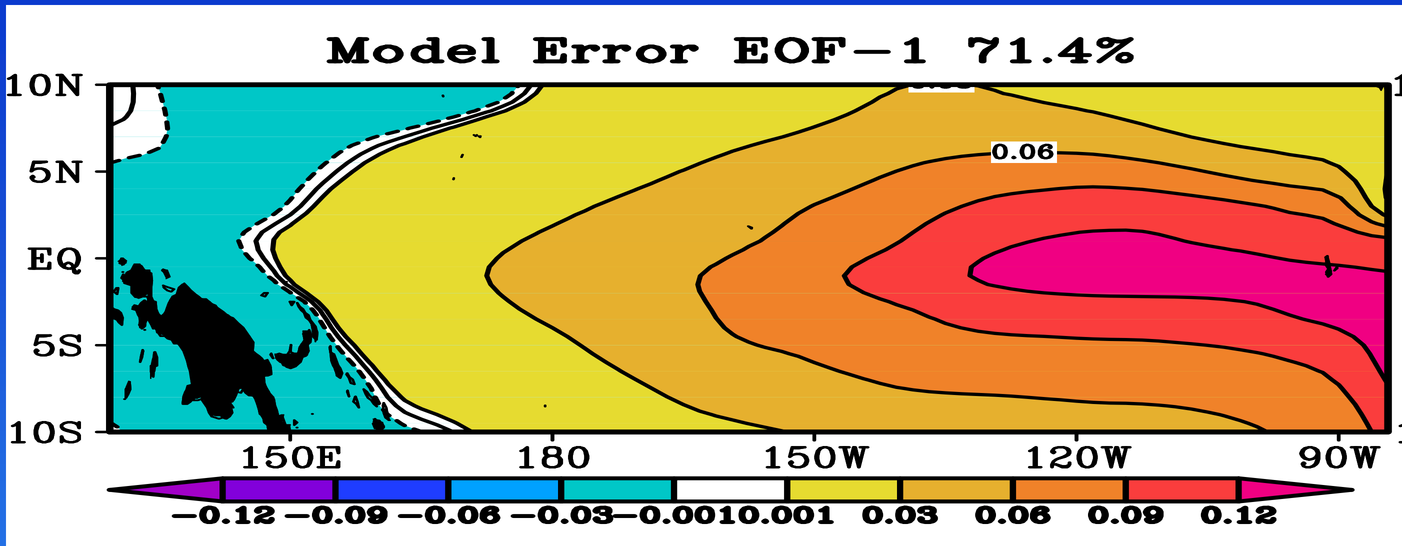


148-year Run

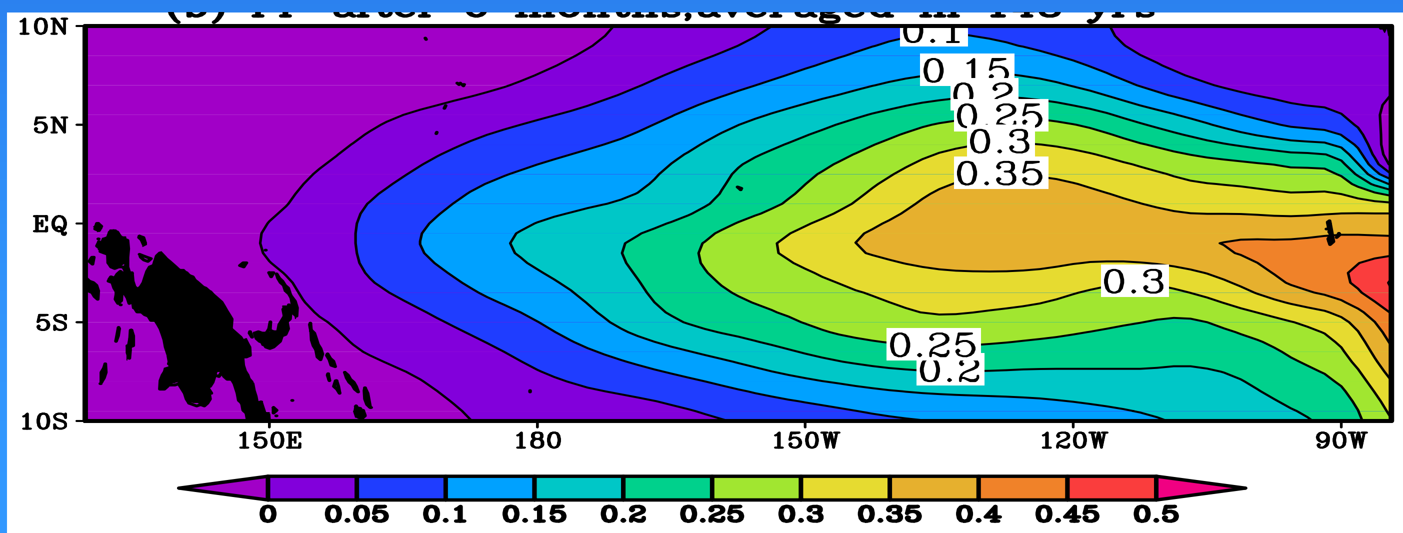


Starting time (calendar month)

EOF-1 for **Real Model Error** at 6-Month Leading Time

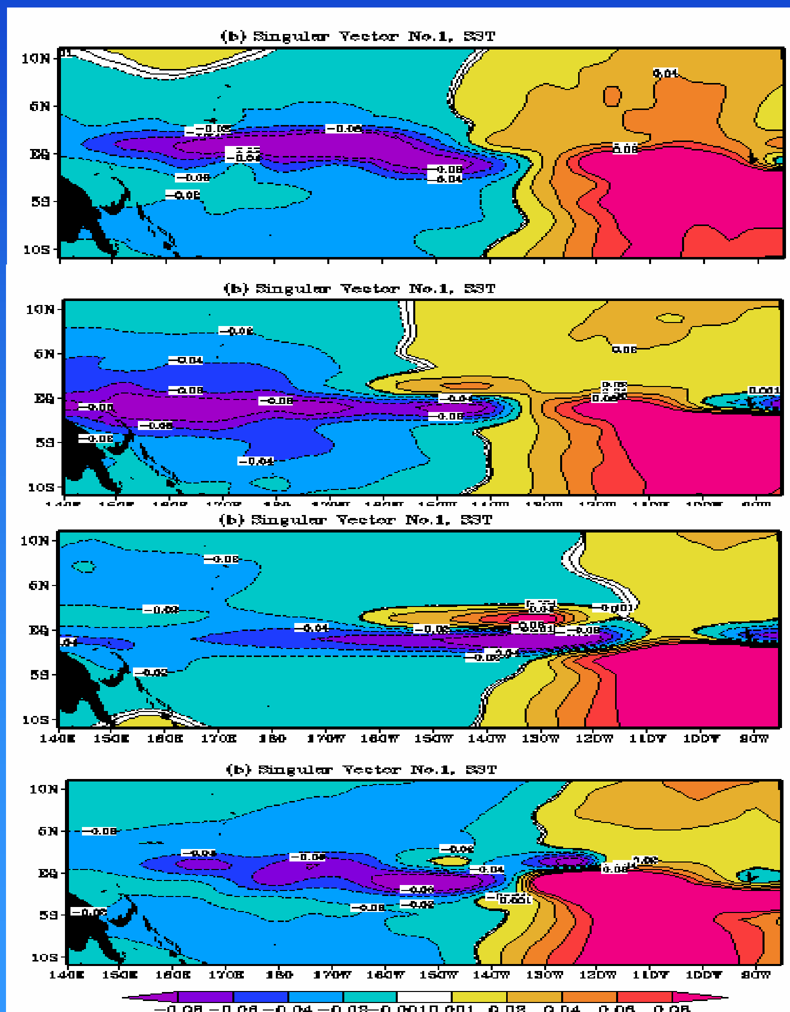


The first **Final pattern** (FP1) at 6-Month Leading Time

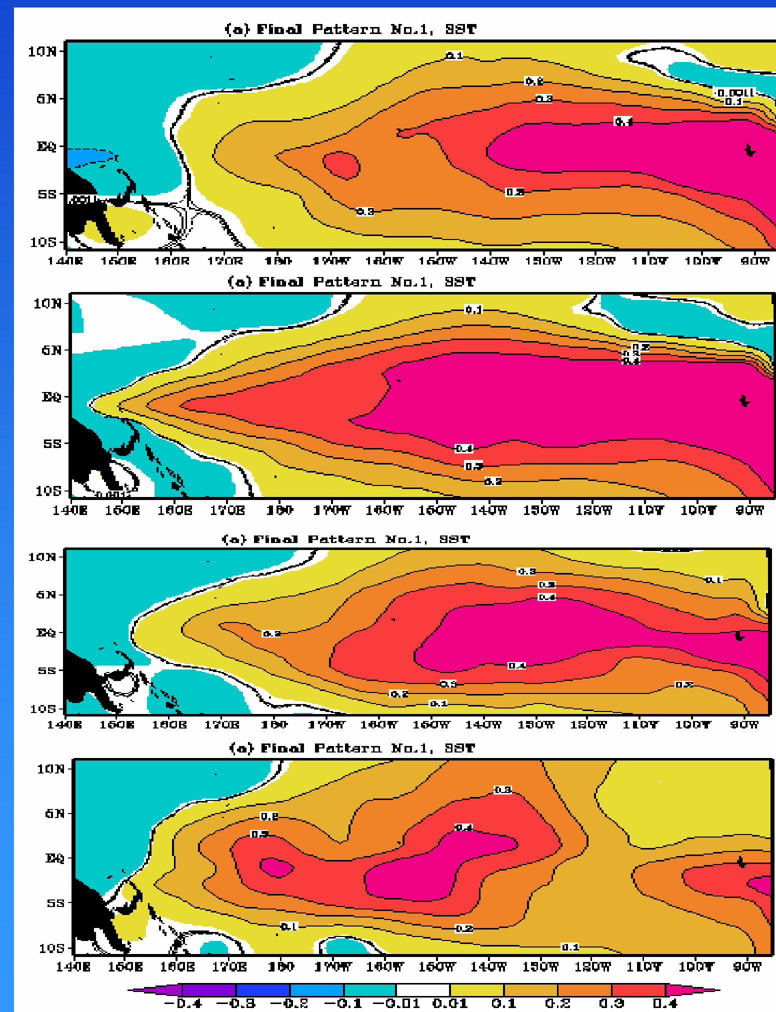


SVs starting at different calendar month

Initial patterns (SVs)



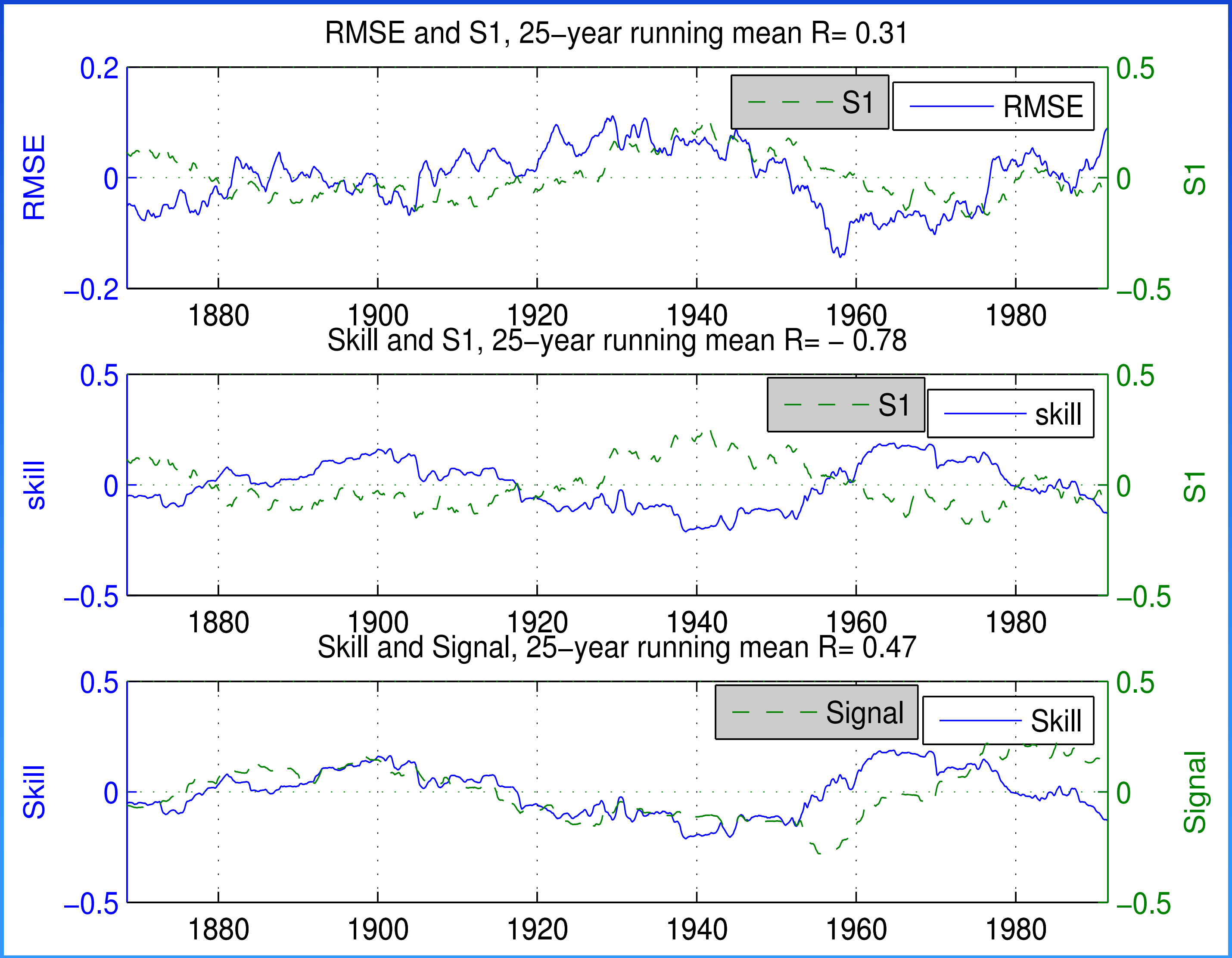
Final Patterns after 6-month



(January, April, July, October)

Interdecadal Variation of RMSE, correlation Skill, Signal, and Singular value (S1)

signal is the variance of Nino3.4 Index.



RMSE & S1

Skill & S1

Skill &signal

Model Error and Linear Heating / Nonlinear Heating

The prognostic equation of SSTA:

$$\frac{\partial T'}{\partial t} = -\bar{u} \cdot T_x' - u'\bar{T}_x - \underline{u'T'_x} - \bar{v}T'_y - v'\bar{T}_y - \underline{v'T'_y}$$

$$- M(\bar{w})T'_z - \left\{M(\bar{w} + w') - M(\bar{w})\right\}\bar{T}_z$$

$$- \underline{\left\{M(\bar{w} + w') - M(\bar{w})\right\}} T_z' - \alpha T'$$

T, u, v, w are SST, zonal, meridional and vertical current velocities, respectively.

The overbar and prime denote the climatological mean and anomalies, respectively.

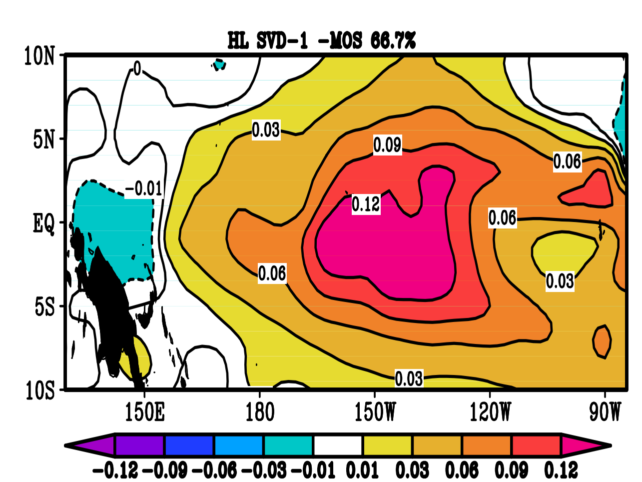
The underline terms are nonlinear heating (NH) terms

SVD analysis for Linear heating

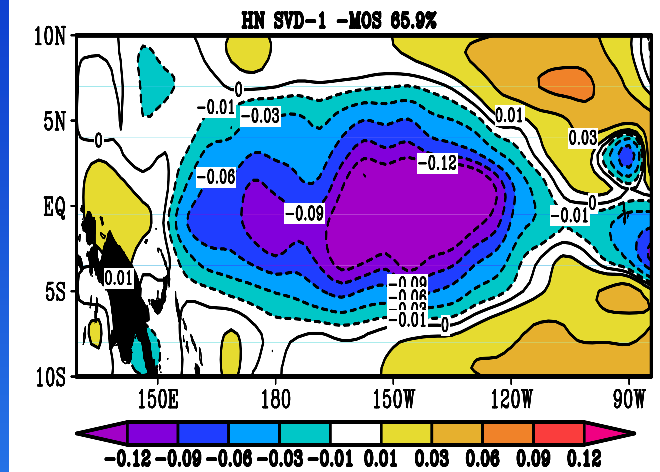
Horizontal (HL) and Vertical (WL)

SVD analysis for Nonlinear heating

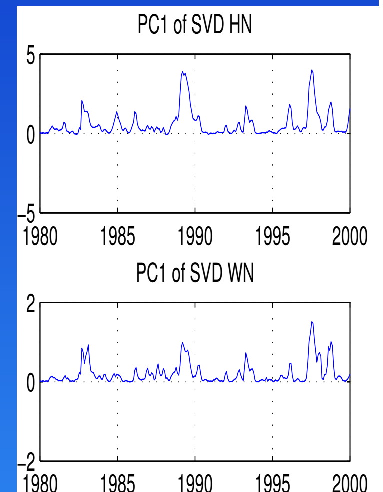
Horizontal (HN) and Vertical (WN)

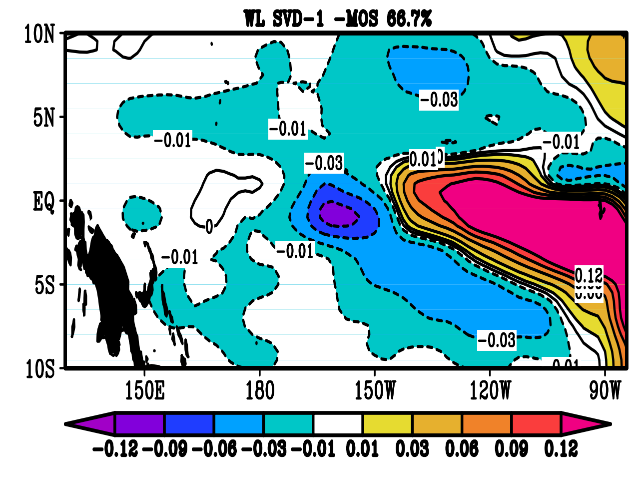


HL

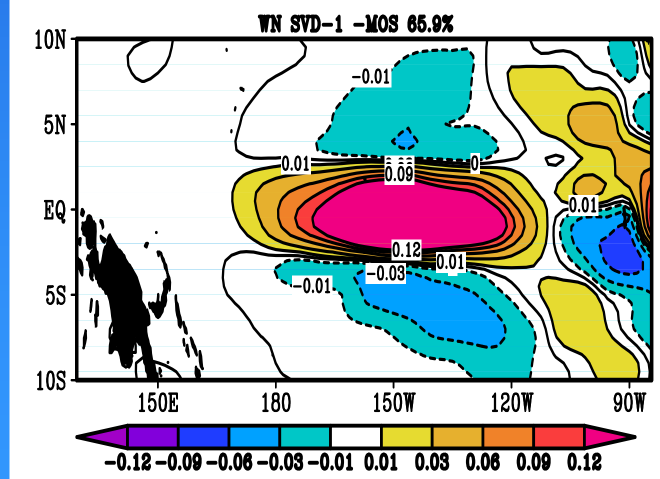


HN



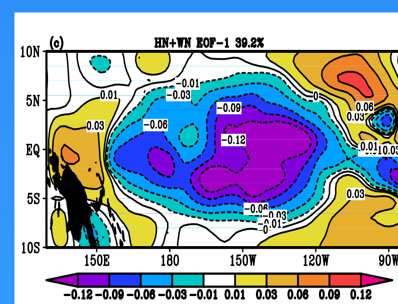


WL



WN

Total Nonlinear



Linear heating

Nonlinear heating

Correlation between Error growth rate (s1) and linear (nonlinear) heating

Linear heating:

Horizontal (HL)

vertical (WL)

Nonlinear heating:

Horizontal (HN)

Vertical (WN)

	Correlation
HL	0.07
WL	-0.35
HN	-0.01
WN	0.07

Linear heating: 80.3% variance

Nonlinear heating: 19.7% variance

Summary

- A tangent linear model is constructed for the LDEO5 model to study the error growth and ENSO predictability for the past 148 years.

- The first SV is a west-east dipole spanned in the equatorial Pacific with one center located in the east and the other in the dateline. It is less sensitive to model initial conditions while there is a strong sensitivity of singular values to initial conditions. Model error grows faster during spring and summer that may be caused by the stronger atmosphere-ocean interaction.

- On the interdecadal time scale, larger model error associated with the faster error growth and lower model skill.

- In ZC model, the nonlinear horizontal advection term leads to a cooling effect and vertical advection always brings a warming effect in the central Pacific Ocean. Is this a model bias? Further analysis is needed to investigate the nonlinear terms.

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References:

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Chen, Y-Q., D.S. Battisti, R.N. Palmer, J. Barsugli, and E. Sarachik, 1997: A study of the predictability of tropical Pacific SST in a coupled atmosphere/ocean model using singular vector analysis. *Mon. Weath. Rev*., 125: 831-845.

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Singular vector analysis

$$\frac{\partial X}{\partial t} = LX$$

$$X(t+\Delta t) = R(t,\Delta t)X(t)$$

$$R(t,\Delta t) = \exp\left(\int_t^{t+\Delta t} Ldt\right)$$

- *L is a matrix which represents the linearized model dynamical operator X denote as the perturbation of model state vectors.*

The amplitude of perturbation growth is:

$$A = \frac{\left\|X(t+\Delta t)\right\|}{\left\|X(t)\right\|} = \frac{\left\langle X(t+\Delta t), X(t+\Delta t)\right\rangle^{1/2}}{\left\langle X(t), X(t)\right\rangle^{1/2}}$$

$$= \frac{\left\langle RX(t), RX(t)\right\rangle^{1/2}}{\left\langle X(t), X(t)\right\rangle^{1/2}} = \frac{\left\langle X(t), R^{*}RX(t)\right\rangle^{1/2}}{\left\langle X(t), X(t)\right\rangle^{1/2}}$$

singular value decomposition (SVD) analysis $R = F\Lambda E^{*}$

E and *F* are singular vectors and final patterns