Assimilation of Data Into Ocean Models:

On-Line Estimation of Background Error Covariance Parameters

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State and Parameter Estimation

Let *x* and *y* be true ocean state and observation vectors, and θ a vector of uncertain parameters of covariance of x. The posterior pdf of state and parameters given observations is

$$p(x,\theta \mid y) \propto p(y \mid x)\, p(x \mid \theta)\, p(\theta)$$

Under the Gaussian error assumption, maximizing the posterior pdf is the same as minimizing L wrt x and θ

$$J \equiv (y - Hx)^T R^{-1} (y - Hx) + (x_b - x)^T B^{-1} (x_b - x)$$

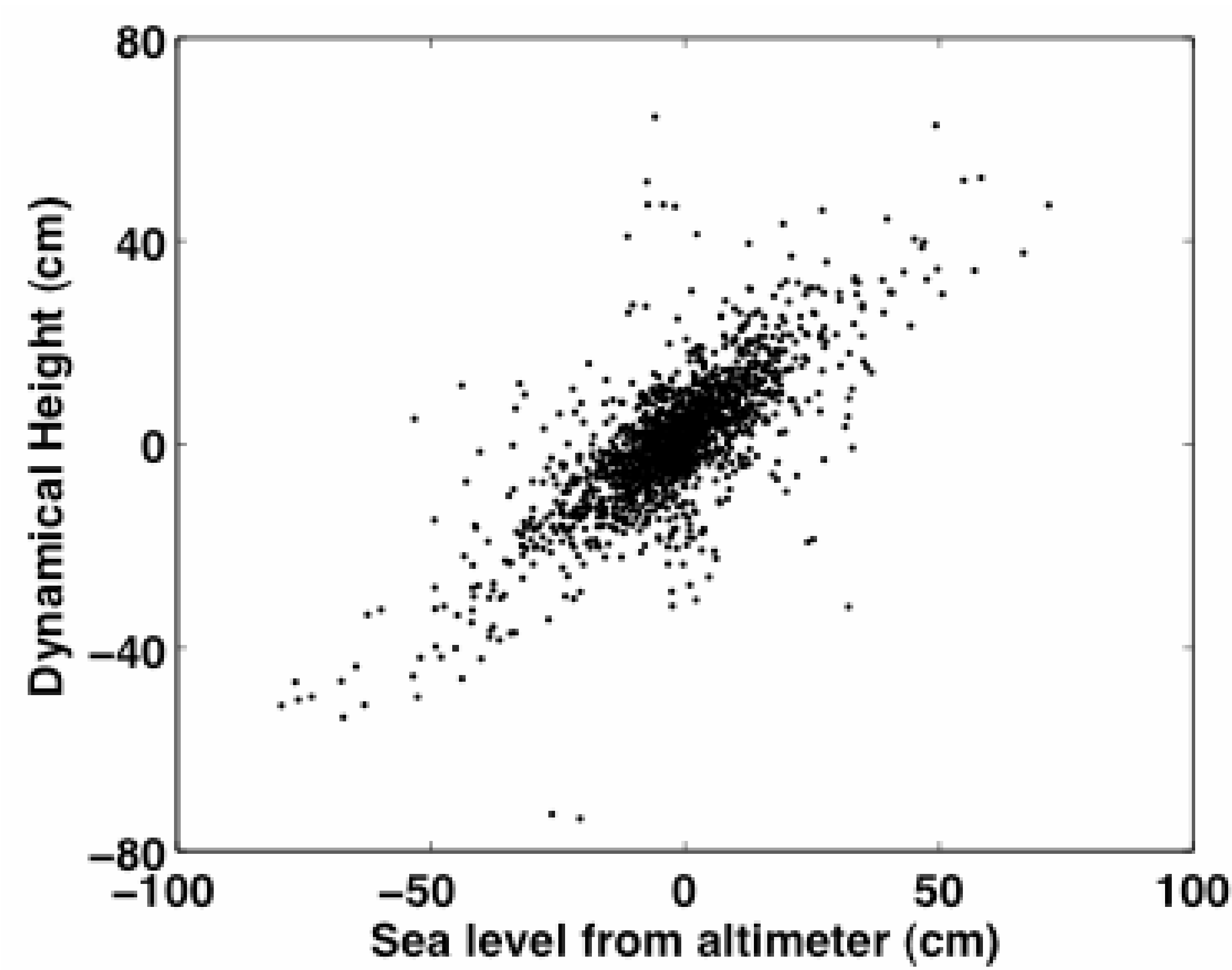
$$L(x,\theta) \propto \log|B(\theta)| + J(x,\theta) - 2\log p(\theta)$$

Online estimation of θ similar in principle to Dee (1995).

Features of the Scheme

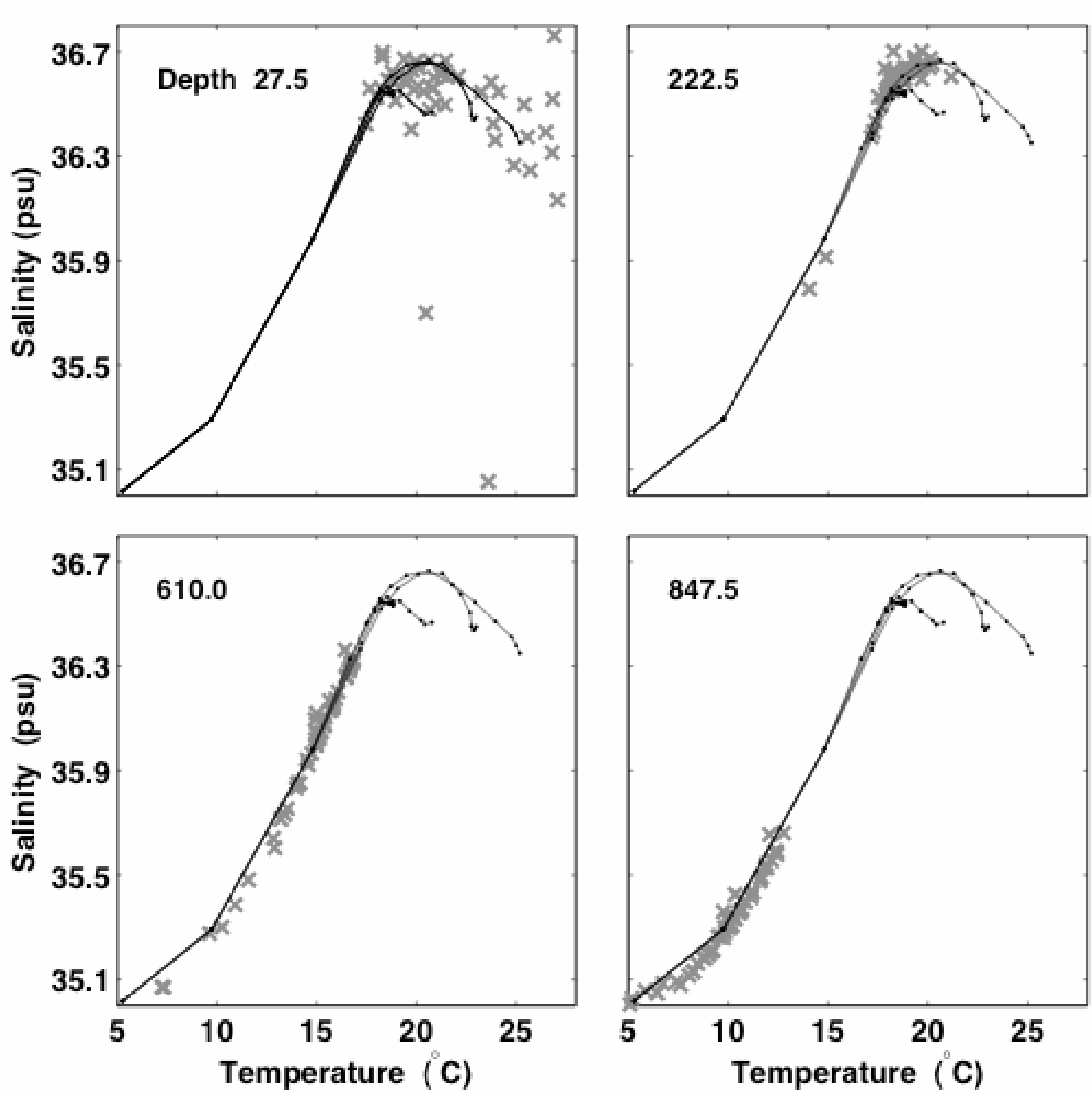
- Extended version of 3DVar.
- Physical constraints used to give B matrix that is physically realistic, state dependent, and easier to deal with computationally.
- Second minimization gives state dependent parameters of B.
- Parameters estimated by maximizing the joint rather than marginal pdf (as in Dee, 1995).

Observed Sea Level and Dynamic Height From Argo



- NW Atlantic
- Anomalies
- Colocated data
- RL at 1160m
- Correlation is 0.75, slope close to 1
- Simple physical balance

Argo Temperature and Salinity



- *Scatterplots of T and S at different depths*
- *~55.2W, 38.4N*
- *Complex, depth dependent structure*
- *Lines show Yashayaev climatology*
- *Shows importance of vertical advection in at depth*

Modelling Uncertainty in the Background State

Motivated by these physical balances, assume

$$T = T_b - \partial T_b / \partial z \xi_D + \xi_T$$

$$S = S_b - \partial S_b / \partial z \xi_D + \xi_S$$

$$\eta = \eta_b + \Delta_\rho \xi_D + \int \left(\alpha_T \xi_T + \alpha_S \xi_S \right) dz$$

Builds on: ***Cooper and Haines (1996), Troccoli and Haines (1999), Haines et al. (2006), Ricci et al. (2005), Weaver et al. (2006)***

Parameterization of B

Physical constraints give state dependent transformation from original to auxiliary variables. Thus B can be written as function of covariance matrix of auxiliary variables.

Assume

$$B_{\zeta\zeta} = B_{DD} \oplus (B_{HH} \otimes B_{VV})_{\varsigma_T, \varsigma_S}$$

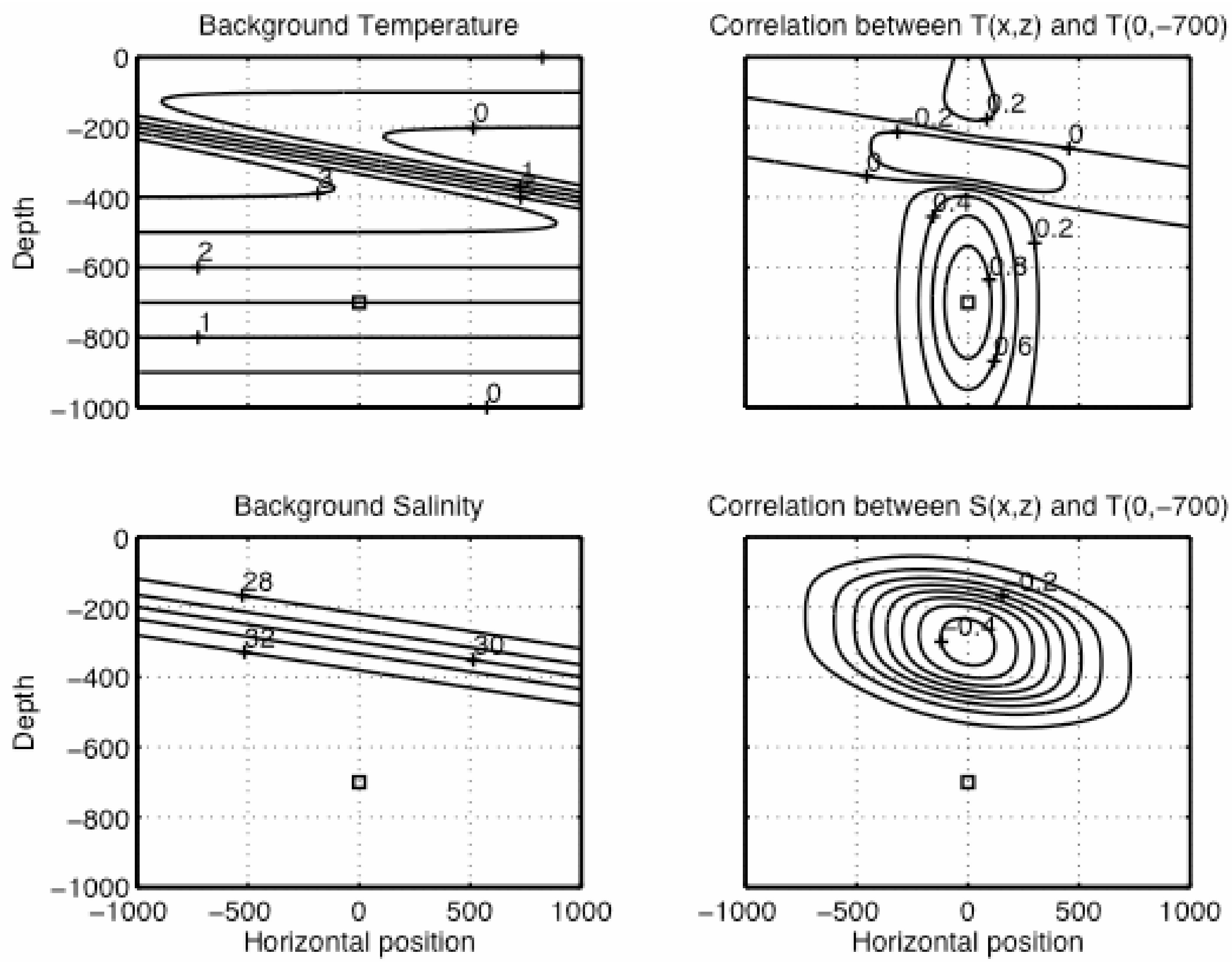
$$B_{VV} = \begin{bmatrix} B_{TT} & B_{TS} \\ B_{TS}^{T} & B_{SS} \end{bmatrix}$$

$$B_{DD} = \gamma_1 C^T \exp(-r^2 L_D^{-2}) \mathrm{C}$$

$$B_{HH} = \gamma_2 C^T \exp(-r^2 L_H^{-2}) C$$

Constructing an Idealized B Matrix

Assume xi are uncorrelated with separable (x,z) covariance:



- **Complex T, S covariance**
- **Depends on background**

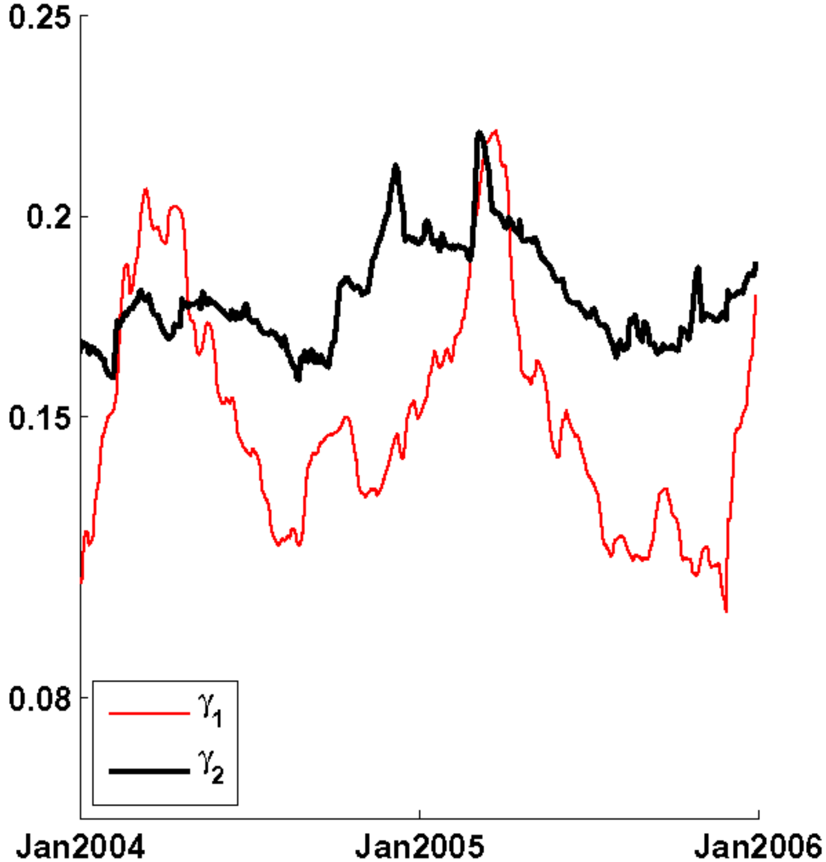
North Atlantic Example

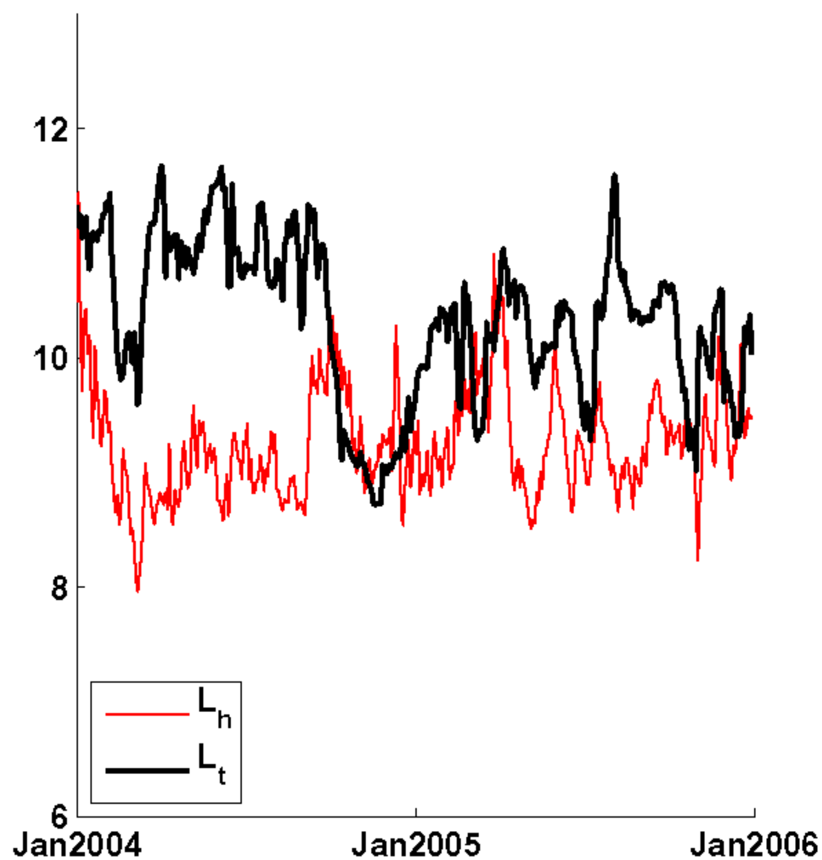
- POP ocean model, 1/3 degree, 23 levels.
- Spectrally nudged to Yashayaev monthly climatology.
- Daily atmospheric forcing from NCEP reanalysis.
- Assimilate Argo and altimeter data, 2003-5.
- Vertical gradient of background is linear combination of climatology and forecast.
- Uncertain covariance parameters (theta) are horizontal length scales and variance of the xi variables.

Time Variation of Parameters

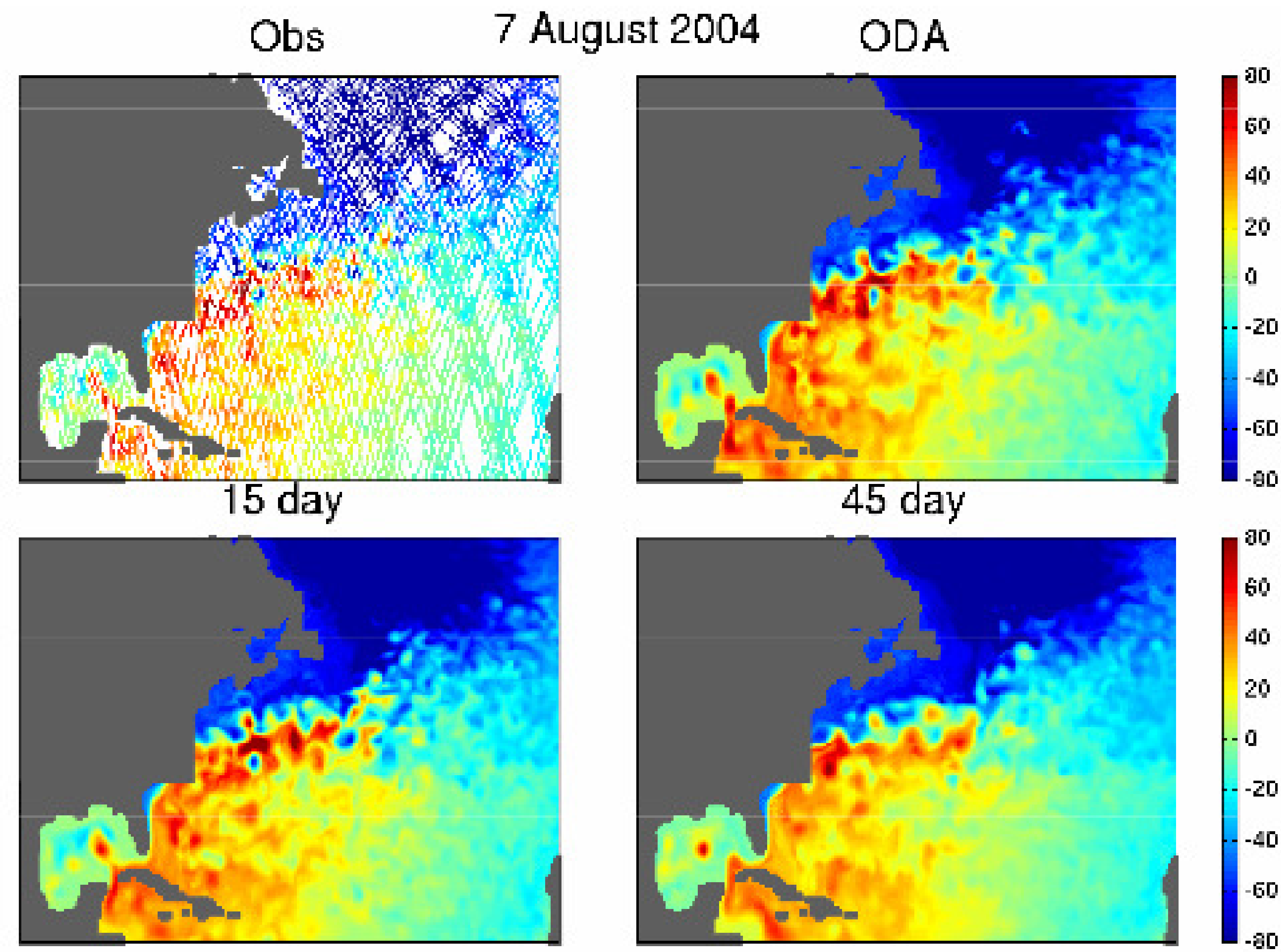
$$B_{DD} = \gamma_1 C^T \exp(-r^2 L_D^{-2}) \mathrm{C}$$

$$B_{HH} = \gamma_2 C^T \exp(-r^2 L_H^{-2}) C$$

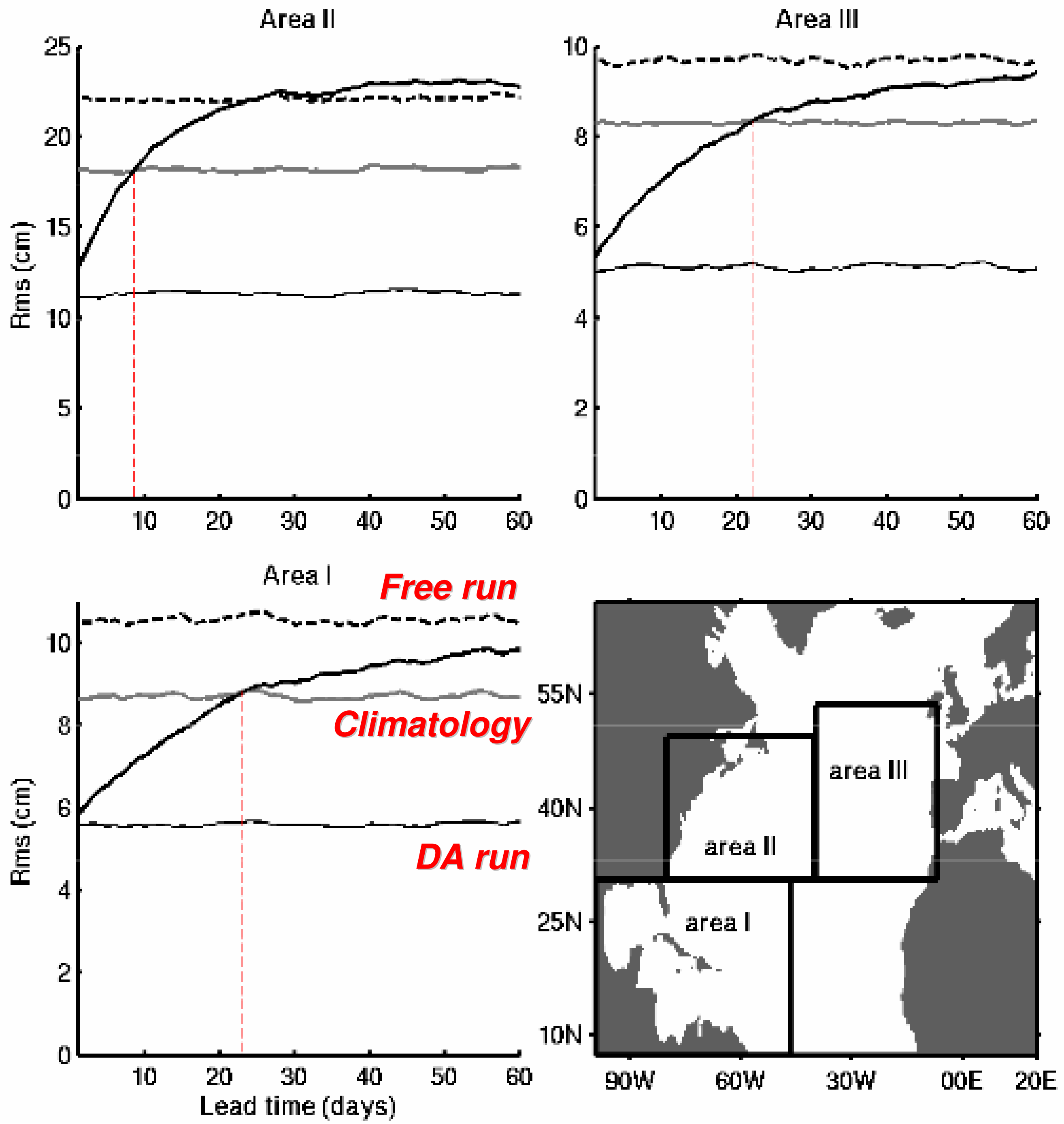




Typical Snapshot of Sea Level

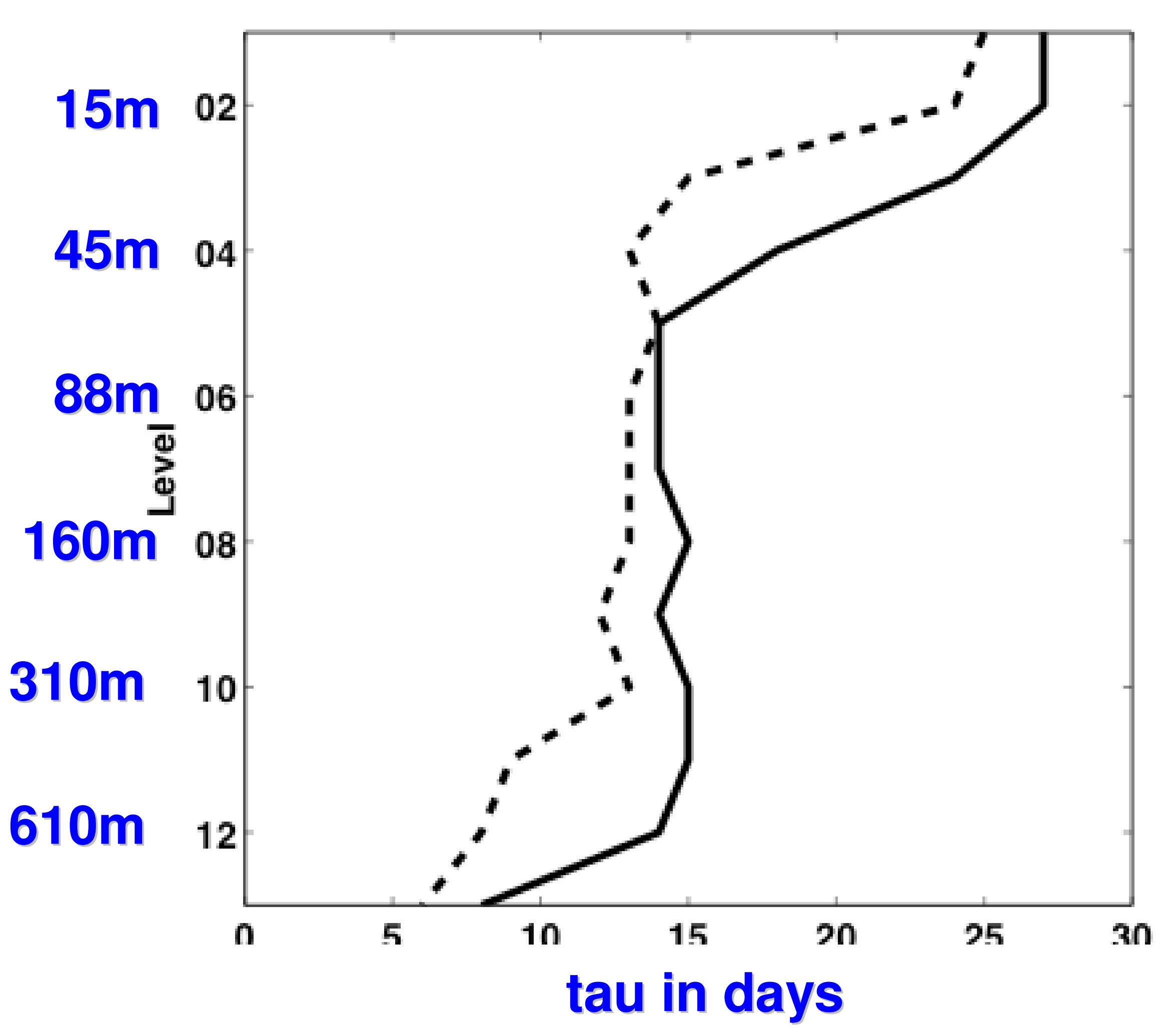


Forecast Skill For Sea Level



- Rms of obs-pred vs lead time
- Based on 24 monthly forecast runs (each 60d)

Forecast Skill for T and S



Summary

- ✓ New scheme is computationally efficient (adds 30% to run time and memory) and has useful skill.
- ✓ Second minimization allows background error covariance matrix (B) to change with state.
- ✓ Online estimation of B gives scheme robustness and flexibility. Computationally feasible because joint posterior pdf maximized rather than marginal.
- ✓ Reasonable parameters estimated every 2 days in the North Atlantic example.
- ✓ Forecasts improved in Gulf Stream region by allowing covariance parameters to change with time.
- ✓ Implemented in NEMO and evaluation underway.