Tangent Linear Approximation of Ocean Mixed Layer Model

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Motivation:

Vertical mixing (eddy viscosity and diffusivity) terms are highly nonlinear and complicated formulation in the general ocean circulation model.

For the development of 4DVAR, we need to

Question:

Can we find a relatively simple formulation of tangent linearized vertical mixing term?

Background

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K(T)\frac{\partial T}{\partial z} \right)$$

: vertical diffusion equation for T

$$T^{(n)} = T^{(n-1)} + \delta T$$

: perturbation in model trajectory

$$\frac{\partial T^{(n)}}{\partial t} = \frac{\partial}{\partial z}\left(K(T^{(n)})\frac{\partial T^{(n)}}{\partial z} + \frac{\partial K(T^{(n-1)})}{\partial T}\frac{\partial T^{(n-1)}}{\partial z} T^{(n)} \right) - f^{(n-1)}$$

: tangent linear (TL) equation
(diffusion-advection type)

Change of type of equation due to the linearization introduces tighter CFL condition to TL equation (Zhu and Kamachi, 2000, MWR).

Derivation of linearized mixing coefficient is not easy

Background (cont'd)

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K(T) \frac{\partial T}{\partial z} \right)$$: vertical diffusion equation for T

$$\frac{\partial T^{(n)}}{\partial t} = \frac{\partial}{\partial z}\left(K(T^{(n-1)}) \frac{\partial T^{(n)}}{\partial z} + \frac{\partial K(T^{(n-1)})}{\partial T} \frac{\partial T^{(n-1)}}{\partial z} T^{(n)} \right) - f^{(n-1)}$$

$$\frac{\partial T^{(n)}}{\partial t} = \frac{\partial}{\partial z}\left(K(T^{(n-1)}) \frac{\partial T^{(n)}}{\partial z} \right)$$: simplified TL equation

Simplified TL equation is numerically stable (Mahfouf, 1999, Tellus), but inaccurate (Zhu and Kamachi, 2000, MWR).

But what is important for TL equation is if $T^{(n)} \rightarrow T \quad \text{as} \quad \text{n} \rightarrow \infty$ (Bennett, 1992;2000).

Ocean Mixed Layer Model

Ocean Model: 1D configuration of OPA9.0/NEMO

$$\frac{\partial U}{\partial t} = fV + \frac{\partial}{\partial z}\left(K_M \frac{\partial U}{\partial z}\right), \quad \frac{\partial V}{\partial t} = -fU + \frac{\partial}{\partial z}\left(K_M \frac{\partial V}{\partial z}\right)$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial \theta}{\partial z}\right), \quad \frac{\partial S}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial S}{\partial z}\right)$$

$$K_M = \frac{K_{\max}}{(1+\alpha \mathrm{Ri})^2}, \quad K_H = \frac{K_M}{1+\alpha \mathrm{Ri}} + \kappa_H$$

Boundary conditions:

- zonal wind stress = const., meridional wind stress = 0
- total heat flux = const., fresh water flux = 0

Resolution:

- 1m within the first 20 levels, 500m at the bottom (50 levels)

Numerical Solution of nonlinear equation

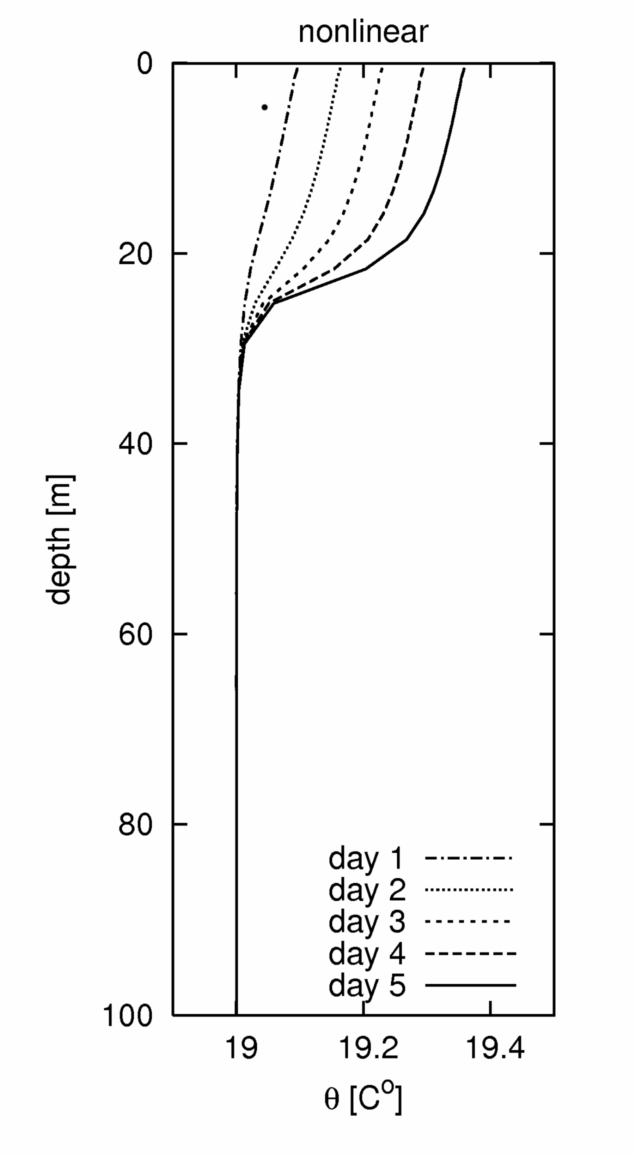
time step size: 120sec

integration period: 5days

initial condition:
  potential temperature 19C
  salinity 35PSU
  rest

Control parameters:

  zonal wind stress: 0.1 N/m2
  heat flux : 70 W/m2



Picard Iteration

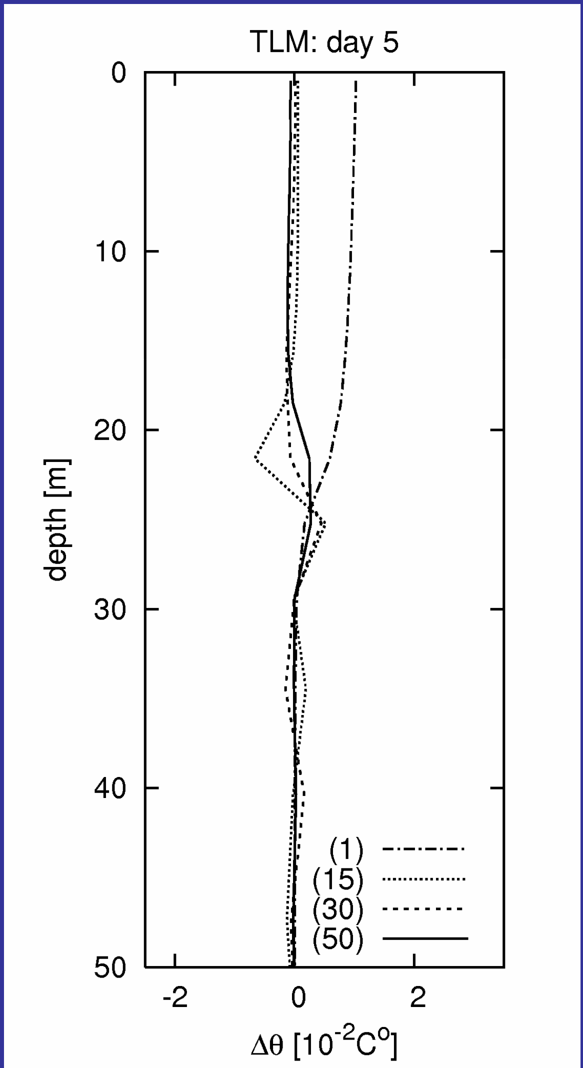
$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K(T)\frac{\partial T}{\partial z} \right)$$: the first guess

$$\frac{\partial T^{(n)}}{\partial t} = \frac{\partial}{\partial z}\left(K(T^{(n-1)})\frac{\partial T^{(n)}}{\partial z} + \frac{\partial K(T^{(n-1)})}{\partial T}\frac{\partial T^{(n-1)}}{\partial z} T^{(n)} \right) - f^{(n-1)}$$

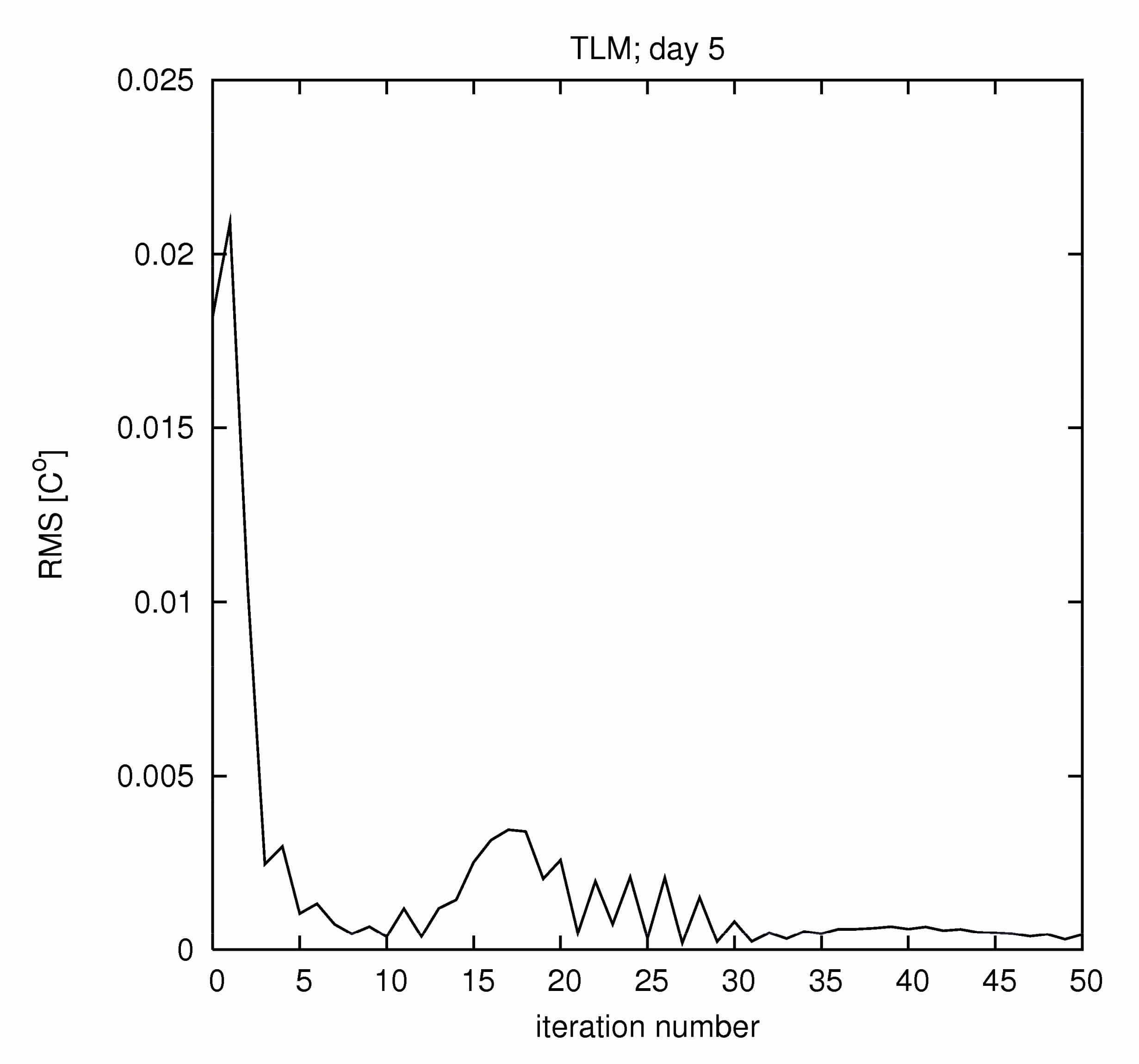
$$\frac{\partial T^{(n)}}{\partial t} = \frac{\partial}{\partial z}\left(K(T^{(n-1)})\frac{\partial T^{(n)}}{\partial z} \right) - f^{(n-1)}$$: TL approximation

Starting from the nonlinear trajectory for the perturbed control parameters as the initial guess (0th solution), substitute the (n-1)th solution to the linearized operator subsequently.

Picard Iteration



anomaly from nonlinear solution



RMS anomaly over the first 20 levels

Bug report

locations: dynzdf.F90, trazdf.F90, trdmod.F90 /NEMO
symptom: time step size is not specified in time-splitting scheme of vertical physics.

```
if(istp == nit000) then
   p2dt = rdt                       ! Euler-Forward
else if(istp <= nit000+1) then
   p2dt = 2.0*rdt                   ! Leap-Frog
endif
```

↓

```
if(istp == nit000) then
   p2dt = rdt                       ! Euler-Forward
else
   p2dt = 2.0*rdt                   ! Leap-Frog
endif
```

Concluding Remarks

Although inaccurate at the first step, the simplified TL equation of vertical diffusion equation approximately converges to the nonlinear solution in the Picard iteration.

The simplified TL is useful for constructing 4DVAR system and this make it easier to develop adjoint code.

Acknowledgment

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